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A COMPARATIVE STUDY OF THE NUMBER OF HANTAVIRUS PULMONARY SYNDROME (HPS) CASES BY USING CUBIC SPLINE INTERPOLATION

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Abstract

Hantavirus pulmonary syndrome (HPS) is a rodent-borne viral disease that was first recognized in 1993 when an outbreak of severe respiratory illnesses occurred among residents of the south-western United States (US). Nationwide, the case-fatality ratio for HPS during 2009-2012 was 37.2 percent. In this manuscript, our aim is to compare the number of HPS cases by using cubic spline interpolation for more accurate result. For this purpose, here we construct the cubic spline for clamped conditions to predict number of HPS cases for a specific year and then compare the result which is recorded from field survey. To solve the simultaneous system of equations, we use MAPLE 13 and to show the result in graph, we use simple bar diagram.

Key words: Hantavirus Pulmonary Syndrome (HPS), cubic spline, interpolation, clamed conditions, compare.

Introduction

The goal of cubic spline is to get an interpolation formula (Burden *et al*., ???) that is continuous in both the first and second derivatives, both within the intervals and at the interpolating nodes. This will give us a smoother interpolating function (Wang, Kai 2013) by which we can predict or measure any value inside the interval and will get almost accurate results. For the case of Hantavirus pulmonary syndrome (HPS), the number of cases and their fatality ratio are obtained from the key finding and public health messages, centers for disease control and prevention for the year 1993-2015 (CDC, 2016). In the section results and discussion, first we construct a mathematical model for HPS cases by using cubic spline interpolation for clamped condition and then we find out the number of HPS cases for some other years. Finally we analyze the results, give some conclusions, and outline the possible future applications of the work.

Materials and Methods

Cubic Spline: Given a function f defined on $[a, b]$ and a set of nodes $a = x_0 < x_1 < x_2 < \dots\dots\dots < x_n = b$ (S. Pruess, 1993). A cubic spline interpolant S for f is a function that satisfies the following conditions:

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a) S(n) is a cubic polynomial denoted by $S_j(x)$ on the subinterval $[x_j, x_{j+1}]$ for each $j = 0,1,2, \dots \dots, n-1$

b) $s_j(x_j) = f(x_j)$ for $j = 0,1,2, \dots \dots, n-1$

c) $s_{j+1}(x_{j+1}) = x_j(x_{j+1})$ for $j = 0,1,2, \dots \dots, n-2$

d) $s'_{j+1}(x_{j+1}) = s'_j(x_{j+1})$ for $j = 0,1,2, \dots \dots, n-2$

e) $s''_{j+1}(x_{j+1}) = s''_j(x_{j+1})$ for $j = 0,1,2, \dots \dots, n-2$

f) One of the following boundary condition is satisfied:

(i) $s''(x_0) = s''(x_n) = 0$ [Natural boundary]

(ii) $s'(x_0) = f'(x_0)$and$s'(x_n) = f'(x_n)$ [Clamed boundary]

Construction of Cubic Spline:

$$s_j(x) = a_j + b_j(x - x_j) + c_j(x - x_j)^2 + d_j(x - x_j)^3 \tag{A}$$

We have from the condition of cubic spline that, $s_j(x_j) = a_j = f(x_j)$ (1)

Now,$a_{j+1} = s_{j+1}(x_{j+1}) = s_j(x_{j+1})$

$$= a_j + b_j(x_{j+1} - x_j) + c_j(x_{j+1} - x_j)^2 + d_j(x_{j+1} - x_j)^3$$

Suppose, $x_{j+1} - x_j = h_j$ then it reduces to the form

$$a_{j+1} = a_j + b_j h_j + c_j h^2{}_j + d_j h^3{}_j \tag{2}$$

$$s'_j(x) = b_j + 2c_j(x_{j+1} - x_j) + 3d_j(x_{j+1} - x_j)^2 \tag{3}$$

Putting $x = x_j$ in (3) we get

$$s'_j(x_j) = b_j \Rightarrow s'_{j+1}(x_{j+1}) = b_{j+1} \text{ [replacing } j \text{ by } j+1]$$

$$\Rightarrow s'_j(x_{j+1}) = b_{j+1} \; [s'_{j+1}(x_{j+1}) = s'_j(x_{j+1})]$$

$$\Rightarrow b_{j+1} = b_j + 2c_j(x_{j+1} - x_j) + 3d_j(x_{j+1} - x_j)^2 \text{ [using (3)]}$$

$$\Rightarrow b_{j+1} = b_j + 2c_j h_j + 3d_j {h_j}^2$$

Again, $s''_j(x) = 2c_j + 6d_j(x - x_j) \Rightarrow s''_j(x_{j+1}) = 2c_j + 6d_j(x_{j+1} - x_j)$

$$\Rightarrow s''_j(x_{j+1}) = 2c_j + 6d_j h_j \tag{5}$$

Again, $s''_j(x_j) = 2c_j \Rightarrow c_j = \frac{1}{2} s''_j(x_j)$

From (5), we get

$$s''_j(x_{j+1}) = 2 \cdot \frac{1}{2} s''_j(x) + 6d_j h_j \Rightarrow s''_j(x_{j+1}) - s''_j(x_j) = 6d_j h_j$$

$$\Rightarrow d_j = \frac{1}{6h_j}\{s''_j(x_{j+1}) - s''_j(x_j)\}$$

Putting the value of a_j, b_j, c_j, d_j in (A) we get,

$$s_j(x) = s_j(x_j) + s'_j(x_j)(x - x_j) + \frac{1}{2}s''_j(x)(x - x_j)^2 + \frac{1}{6h_j}\{s''_j(x_{j+1}) - s''_j(x_j)\}(x - x_j)^3$$

Governing equations

Suppose a cubic spline polynomial is

$$s_j(x) = a_j + b_j(x - x_j) + c_j(x - x_j)^2 + d_j(x - x_j)^3$$

For $j = 0,1,2, \dots \dots, n - 1$ (1)

Here clearly find $s_j(x_j) = a_j = f(x_j)$ (A)

We have the condition $s_{j+1}(x_{j+1}) = s_j(x_{j+1})$

$\therefore$ from (A) we get, $a_{j+1} = s_{+1}(x_{j+1})$

$$= a_j + b_j(x_{j+1} - x_j) + c_j(x_{j+1} - x_j)^2 + d_j(x_{j+1} - x_j)^3 \quad (2)$$

$$a_{j+1} = a_j + b_j h_j + c_j h^2{}_j + d_j h^3{}_j \quad (3)$$

Differentiating (1) with respect to x

$$s_j{}'(x) = b_j + 2c_j(x - x_j) + 3d_j(x - x_j)^2 \quad (B)$$

$$\Rightarrow s_j{}'(x_j) = b_j + 2c_j(x_{j+1} - x_j) + 3d_j(x_{j+1} - x_j)^2$$

$$\Rightarrow b_{j+1} = b_j + 2c_j h_j + 3d_j h^2{}_j \quad (4)$$

Again differentiating (B) with respect to x

$s_j{}''(x) = 2c_j + 6d_j(x - x_j)$ and $s_j{}''(x_j) = 2c_j \Rightarrow c_j = \frac{s_j{}''(x_j)}{2}$ (C)

$\Rightarrow c_{j+1} = \frac{s_{j+1}{}''(x_{j+1})}{2} = \frac{1}{2}\{2c_j + 6d_j h_j\}$

$\Rightarrow c_{j+1} = c_j + 3d_j h_j, j = 0,1,2, \dots \dots, n - 1$

$\Rightarrow d_j = \frac{1}{3h_j}\{c_{j+1} - c_j\}$

Putting the value of d_j in equation (3)

$$a_{j+1} = a_j + b_j h_j + c_j h^2{}_j + \frac{h^3{}_j}{3h_j}\{c_{j+1} - c_j\} = a_j + b_j h_j + c_j h^2{}_j + \frac{h^2{}_j}{3}\{c_{j+1} - c_j\}$$

$$= a_j + b_j h_j + + \frac{h^2{}_j}{3}\{c_{j+1} + 2c_j\} \quad (5)$$

And from (4) $\Rightarrow b_{j+1} = b_j + 2c_j h_j + 3h^2{}_j \cdot \frac{1}{3h_j}\{c_{j+1} - c_j\} = b_j + h_j\{c_{j+1} + c_j\}$ (6)

Now from (5)$a_{j+1} - a_j = b_j h_j + + \frac{h^2{}_j}{3}\{c_{j+1} + 2c_j\}$

$$\Rightarrow b_j = \frac{1}{h_j}\{(a_{j+1} - a_j) - \frac{h_j{}^2}{3h_j}(2c_j + c_{j+1})\}$$

$$\Rightarrow b_j = \frac{1}{h_j}(a_{j+1} - a_j) - \frac{h_j}{3}(2c_j + c_{j+1}) \tag{7}$$

Putting $j = j - 1$ in the above equation and in (6)

$$b_{j-1} = \frac{1}{h_{j-1}}\{(a_j - a_{j-1}) - \frac{h_{j-1}}{3}(2c_{j-1} + c_j)\} \tag{8}$$

And $b_j = b_{j-1} + h_{j-1}\{c_j + c_{j-1}\}$

$\Rightarrow \frac{1}{h_j}(a_{j+1} - a_j) - \frac{h_j}{3}(2c_j + c_{j+1}) = \frac{1}{h_{j-1}}(a_j - a_{j-1}) - \frac{h_{j-1}}{3}(2c_{j-1} + c_j) + h_{j-1}(c_j +$
$c_{j-1})$

$$\Rightarrow \frac{3}{h_j}(a_{j+1} - a_j) - \frac{3}{h_{j-1}}(a_j - a_{j-1})$$

$$= h_j(2c_j + c_{j+1}) - h_{j-1}(2c_{j-1} + c_j) + 3h_{j-1}(c_j + c_{j-1})$$

$$\Rightarrow \frac{3}{h_j}(a_{j+1} - a_j) - \frac{3}{h_{j-1}}(a_j - a_{j-1}) = h_j(2c_j + c_{j+1}) + h_{j-1}(2c_j + c_{j-1})$$

$$\Rightarrow \frac{3}{h_j}(a_{j+1} - a_j) - \frac{3}{h_{j-1}}(a_j - a_{j-1}) = 2c_j(h_j + h_{j-1}) + h_{j-1}c_{j-1} + h_j c_{j+1}$$

This system involves only $\{c_j\}_j^n = 0$ as unknown since the values of $\{h_j\}_j^{n-1} = 0$ and $\{a_j\}_j^n = 0$ are given by the spacing of the nodes $\{x_j\}_j^n = 0$ and the values of f at the nodes.

Theorem: If f is defined at $a = x_0 < x_1 < \cdots < x_n = b$ and differentiable at a and b then f has a unique clamed Spline interpolant on the nodes $x_0, x_1, x_2, \dots \dots .., x_n$ that is a spline interpolant that satisfies the boundary conditions $s'(a) = f'(a)$ and $s'(b) = f'(b)$.

Proof: It can be seen, using the fact that

$s'(a) = s'(x_0) = b_0$. Now we have, $f'(a) = \frac{a_1 - a_0}{h_0} - \frac{h_0}{3}(2c_0 + c_1)$

Consequently, $2h_0c_0 + h_0c_1 = \frac{3}{h_0}(a_1 - a_0) - 3f'(a)$

Similarly, $f'(b) = b_n = b_{n-1} + h_{n-1}(c_{n-1} + c_n))$ (A)

Now we have the equation, $b_j = \frac{1}{h_j}(a_{j+1} - a_j) - \frac{h_j}{3}(2c_j + c_{j+1})$

Putting $j = n - 1$ in the above equation we have,

$$b_{n-1} = \frac{1}{h_{n-1}}(a_n - a_{n-1}) - \frac{h_{n-1}}{3}(2c_{n-1} + c_n)$$

$$f'(b) = \frac{1}{h_{n-1}}(a_n - a_{n-1}) - \frac{h_{n-1}}{3}(2c_{n-1} + c_n) + h_{n-1}(c_{n-1} + c_n)$$

$$= \frac{a_n - a_{n-1}}{h_{n-1}} + \frac{h_{n-1}}{3}(c_{n-1} + 2c_n)$$

And $h_{n-1}c_{n-1} + 2h_{n-1}c_n = 3f'(b) - \frac{3}{h_{n-1}}(a_n - a_{n-1})$.

Also $2h_0c_0 + h_0c_1 = \frac{3}{h_0}(a_1 - a_0)3f'(a)$.

Then we determine the linear system $Ax = b$ where

$$A = \begin{bmatrix} 2h_0 & h_0 & 0 & \cdots & \cdots & 0 \\ h_0 & 2(h_0+h_1) & h_1 & \cdots & \cdots & \vdots \\ 0 & h_1 & 2(h_1+h_2) & h_2 & \cdots & \vdots \\ \vdots & \cdots & \cdots & \cdots & \cdots & 0 \\ \vdots & \cdots & \cdots & h_{n-2} & 2(h_{n-2}+h_{n-1}) & h_{n-1} \\ 0 & 0 & 0 & 0 & h_{n-1} & 2h_{n-1} \end{bmatrix}$$

$$b = \begin{bmatrix} \frac{3}{h_0}(a_1 - a_0) - 3f'(a) \\ \frac{3}{h_0}(a_2 - a_1) - \frac{3}{h_0}(a_1 - a_0) \\ \vdots \\ \vdots \\ \frac{3}{h_{n-1}}(a_n - a_{n-1}) - \frac{3}{h_{n-2}}(a_{n-1} - a_{n-2}) \\ 3f'(b) - \frac{3}{h_{n-1}}(a_n - a_{n-1}) \end{bmatrix} \quad \text{and} \quad c = [c_0 \quad c_1 \quad \cdots \quad \cdots \quad c_{n-1} \quad c_n]^T$$

Results and Discussion

Let us consider the annual U.S. HPS cases that are given bellow in random basis from the year 1993-2015. Here we will use cubic spline to interpolate the number of HPS cases for different years:

Time	1993	1995	1998	2002	2005	2008	2010	2012	2013	2015
HPS Case	48	24	33	23	34	24	21	29	21	18
Case Fatality $f'(x)$	27	42	27	43	29	50	29	41	43	22

First of all, to interpolate the number of HPS cases for different years, we need to model a cubic spline interpolation for this data. For this purpose, let us consider the time by the variable x and the corresponding HPS case by a. For calculation, let us consider the initial time 1993 as $t = 0$, and then 1995 as $t = 2$ and so on. Then the nodes are $x_0 = 0, x_1 = 2, x_2 = 5, x_3 = 9, x_4 = 12, \quad x_5 = 15, x_6 = 17, x_7 = 19, x_8 = 20, x_9 = 22$ ...

Since, $a_j = f(x_j) = s_j(x_j)$ we get,

$a_0 = 48, a_1 = 24, a_2 = 33, a_3 = 23, a_4 = 34, \quad a_5 = 24, a_6 = 21, a_7 = 29, a_8 = 21, a_9 = 18$

Also, $f'(x_j) = b_j$ given, $f'(x_0) = b_0 = 27$ and $f'(x_9) = b_9 = 22$

From the relation, $h_j = x_{j+1} - x_j$ we find

$$h_0 = 2, h_1 = 3, h_2 = 4, h_3 = 3, h_4 = 3, \quad h_5 = 2, h_6 = 2, h_7 = 1, h_8 = 2$$

To find the value of $\{c_j\}_j^n = 0$ in case of clamed Cubic Spline interpolation, we have the (Rana, S. S ,1990) linear system $A\bar{x} = \bar{b}$ where

$$A = \begin{bmatrix} 2h_0 & h_0 & 0 & \cdots & \cdots & 0 \\ h_0 & 2(h_0+h_1) & h_1 & \cdots & \cdots & \vdots \\ 0 & h_1 & 2(h_1+h_2) & h_2 & \cdots & \vdots \\ \vdots & \cdots & \cdots & \cdots & \cdots & 0 \\ \vdots & \cdots & \cdots & h_{n-2} & 2(h_{n-2}+h_{n-1}) & h_{n-1} \\ 0 & 0 & 0 & 0 & h_{n-1} & 2h_{n-1} \end{bmatrix}$$

$$b = \begin{bmatrix} \frac{3}{h_0}(a_1 - a_0) - 3f'(a) \\ \frac{3}{h_0}(a_2 - a_1) - \frac{3}{h_0}(a_1 - a_0) \\ \vdots \\ \vdots \\ \frac{3}{h_{n-1}}(a_n - a_{n-1}) - \frac{3}{h_{n-2}}(a_{n-1} - a_{n-2}) \\ 3f'(b) - \frac{3}{h_{n-1}}(a_n - a_{n-1}) \end{bmatrix} \quad \text{and} \quad x = \begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ \vdots \\ c_{n-1} \\ c_n \end{bmatrix}$$

The system $A\bar{x} = \bar{b}$ becomes

$$\begin{bmatrix} 4 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 10 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 14 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 14 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 12 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 10 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 & 8 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 6 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 6 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \end{bmatrix} = \begin{bmatrix} -117 \\ 45 \\ -1.5 \\ 1.5 \\ 1 \\ -14.5 \\ 7.5 \\ 12 \\ -19.5 \\ 70.5 \end{bmatrix}$$

which gives

$$\begin{aligned}
4c_0 + 2c_1 &= -117\\
2c_0 + 10c_1 + 3c_2 &= 45\\
3c_1 + 14c_2 + 4c_3 &= -1.5\\
4c_2 + 14c_3 + 3c_4 &= 1.5\\
3c_3 + 12c_4 + 3c_5 &= 1\\
3c_4 + 10c_5 + 5c_6 &= -14.5\\
5c_5 + 8c_6 + 2c_7 &= 7.5\\
2c_6 + 6c_7 + c_8 &= 12\\
c_7 + 6c_8 + 2c_9 &= -19.5\\
2c_8 + 4c_9 &= 70.5
\end{aligned}$$

By using MAPLE 13, we have the following results,

$c_0 = -35.506554$, $c_1 = 12.513108$, $c_2 = -3.039325$, $c_3 = 0.877807$, $c_4 = 0.4559988$,
$c_5 = -2.368469$, $c_6 = 1.563340$, $c_7 = 3.417813$, $c_8 = -11.63356$, $c_9 = 23.44178$

Now, we know $c_{j+1} = c_j + 3d_j h_j$, this implies $d_j = \frac{1}{3h_j}(c_{j+1} - c_j)$ and then we get the followings:

$d_0 = \frac{1}{3h_0}(c_1 - c_0) = -8.003277$, $d_1 = \frac{1}{3h_1}(c_2 - c_1) = -1.72804$, $d_2 = \frac{1}{3h_2}(c_3 - c_2) = 0.4896415$,

$d_3 = \frac{1}{3h_3}(c_4 - c_3) = -0.046867$, $d_4 = \frac{1}{3h_4}(c_5 - c_4) = -0.313829$, $d_5 = \frac{1}{3h_5}(c_6 - c_5) = 0.65530$

$d_6 = \frac{1}{3h_6}(c_7 - c_6) = 0.309078$, $d_7 = \frac{1}{3h_7}(c_8 - c_7) = -5.017124$, $d_8 = \frac{1}{3h_8}(c_9 - c_8) = 5.845890$

Also we have the relation $b_j = \frac{1}{h_j}(a_{j+1} - a_j) - \frac{h_j}{3}(2c_j + c_{j+1})$

$\therefore b_1 = \frac{1}{h_1}(a_2 - a_1) - \frac{h_1}{3}(2c_1 + c_2) = -18.716891$, $b_2 = \frac{1}{h_2}(a_3 - a_2) - \frac{h_2}{3}(2c_2 + c_3) = 4.434457$,

$b_3 = \frac{1}{h_3}(a_4 - a_3) - \frac{h_3}{3}(2c_3 + c_4) = 1.455053$, $b_4 = \frac{1}{h_4}(a_5 - a_4) - \frac{h_4}{3}(2c_4 + c_5) = -1.876861$,

$b_5 = \frac{1}{h_5}(a_6 - a_5) - \frac{h_5}{3}(2c_5 + c_6) = 0.615732$, $b_6 = \frac{1}{h_6}(a_7 - a_6) - \frac{h_6}{3}(2c_6 + c_7) = -0.362995$,

$b_7 = \frac{1}{h_7}(a_8 - a_7) - \frac{h_7}{3}(2c_7 + c_8) = -6.4006887$, $b_8 = \frac{1}{h_8}(a_9 - a_8) - \frac{h_8}{3}(2c_8 + c_9) = -1.61644$

Thus the spline has the equation $s_j(x) = a_j + b_j(x - x_j) + c_j(x - x_j)^2 + d_j(x - x_j)^3$

where the values of the constants for the specific nodes are as follows:

j	x_j	a_j	b_j	c_j	d_j
0	0	48	27	-35.506554	-8.003277
1	2	24	-18.716891	12.513108	-1.72804
2	5	33	4.434457	-3.039325	0.4896415
3	9	23	1.455053	0.877807	-0.046867
4	12	34	-1.876861	0.4559988	-0.313829
5	15	24	0.615732	-2.368469	0.6553015
6	17	21	-0.362995	1.563340	0.309078
7	19	29	-6.4006887	3.417813	-5.017124
8	20	21	-1.61644	-11.63356	5.845890
9	22	18	22	23.44178	----------

Now, to predict the number of HPS cases in the year 1994, here $t=1$ that lies in the interval [0, 2] i.e. $[x_0, x_1]$

Therefore $s(1) = s_0(1) = a_0 + b_0(1-x_0) + c_0(1-x_0)^2 + d_0(1-x_0)^3$.

$$= 48 + 27(1-0) + (-35.506554)(1-0)^2 + (-8.003277)(1-0)^3$$

$$= 31.48 = 23 \text{ (Approximately)}.$$

To predict the number of HPS cases in the year 1996, here $t=3$ that lies in the interval [2, 5] i.e. $[x_1, x_2]$

Therefore $s(3) = s_0(3) = a_1 + b_1(3-x_1) + c_1(3-x_1)^2 + d_1(3-x_1)^3$.

$$= 24 + (-18.716891)(3-2) + (12.513108)(3-2)^2 + (-1.72804)(3-2)^3$$

$$= 16.068177 = 17 \text{ (Approximately)}.$$

To predict the number of HPS cases in the year 1997, here $t=4$ that lies in the interval [2, 5] i.e. $[x_1, x_2]$

Therefore $s(4) = s_0(4) = a_1 + b_1(4-x_1) + c_1(4-x_1)^2 + d_1(4-x_1)^3$.

$$= 24 + (-18.716891)(4-2) + (12.513108)(4-2)^2 + (-1.72804)(4-2)^3$$

$$= 22.79433 = 23 \text{ (Approximately)}.$$

To predict the number of HPS cases in the year 1999, here $t=6$ that lies in the interval [5,9] i.e. $[x_2, x_3]$

Therefore $s(6) = s_0(6) = a_2 + b_2(6-x_2) + c_2(6-x_2)^2 + d_2(6-x_2)^3$.

$$= 33 + (4.434457)(6-5) + (-3.039325)(6-5)^2 + (0.4896415)(6-5)^3$$

$$= 34.8847735 = 35 \text{ (Approximately)}.$$

To predict the number of HPS cases in the year 2004, here $t = 11$ that lies in the interval [9, 12] i.e. $[x_3, x_4]$

Therefore $s(11) = s_3(11) = a_3 + b_3(11 - x_3) + c_3(11 - x_3)^2 + d_3(11 - x_3)^3$.

$= 23 + (1.455053)(11 - 9) + (0.877807)(11 - 9)^2 + (-0.046867)(11 - 9)^3$

$= 29.046398 = 30$ (Approximately).

To predict the number of HPS cases in the year 2006, here $t = 13$ that lies in the interval

[12, 15] i.e. $[x_4, x_5]$

Therefore $s(13) = s_4(13) = a_4 + b_4(13 - x_4) + c_4(13 - x_4)^2 + d_4(13 - x_4)^3$.

$= 34 + (-1.876861)(13 - 12) + (0.4559988)(13 - 12)^2(-0.313829)(13 - 12)^3$

$= 32.26530 = 33$ (Approximately).

To predict the number of HPS cases in the year 2009, here $t = 16$ that lies in the interval [15, 17] i.e. $[x_5, x_6]$

Therefore $s(16) = s_5(16) = a_5 + b_5(16 - x_5) + c_5(16 - x_5)^2 + d_5(16 - x_5)^3$.

$= 24 + (0.615732)(16 - 15) + (-2.368469)(16 - 15)^2 + (0.6553015)(16 - 15)^3$

$= 22.9025645 = 23$ (Approximately).

To predict the number of HPS cases in the year 2011, here $t = 18$ that lies in the interval

[17, 19] i.e. $[x_6, x_7]$

Therefore $s(18) = s_6(18) = a_6 + b_6(18 - x_6) + c_6(18 - x_6)^2 + d_6(18 - x_6)^3$.

$= 21 + (-0.362995)(18 - 17) + (1.563340)(18 - 17)^2 + (0.309078)(18 - 17)^3$

$= 22.509423 = 23$ (Approximately).

To predict the number of HPS cases in the year 2014, here $t = 21$ that lies in the interval

[20, 22] i.e. $[x_8, x_9]$

Therefore $s(21) = s_8(21) = a_8 + b_8(21 - x_8) + c_8(21 - x_8)^2 + d_8(21 - x_8)^3$.

$= 21 + (-1.61644)(21 - 20) + (-11.63356)(21 - 20)^2 + (5.845890)(21 - 20)^3$

$= 13.5687402 = 14$ (Approximately).

Now we can construct a comparative table as follows:

Year	Calculated HPS Case	Original HPS Case	Difference
1994	23	32	9
1996	22	17	-5
1997	23	23	0
1999	35	43	8
2004	30	31	1
2006	33	41	8
2009	23	20	-3
2011	23	24	1
2014	14	35	21

The comparative graph of calculated HPS case, Original HPS cases and their differences are shown by the following bar diagram:

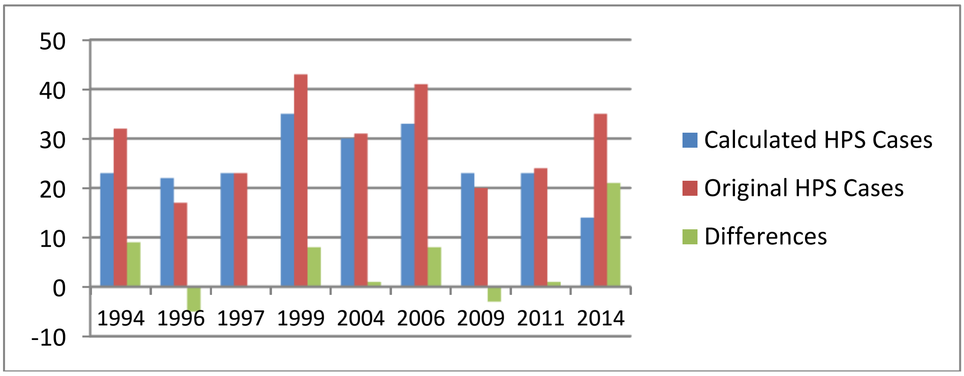

Fig. Compression between calculated HPS and Original HPS cases with their differences

Conclusion

Cubic Spline is more effective for interpolating polynomials especially for piece-wise polynomials than any other interpolation. In this manuscript, we tried to apply the cubic spline interpolation to calculate the number of HPS cases for different years and we see that the result is almost same as they are observed in the field survey. After that prediction, we can conclude that, if the outbreak is continuing by this way then we can say, more accurately, how many would be effected by the coming years. In future, anyone can model this method to interpolate the cases of other virus diseases.

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