ISSN: 3006-7251 (Online)
MBSTU Journal of Science and Technology
DOI: https://doi.org/10.69728/jst.v12.108
Journal Home page: https://journal.mbstu.ac.bd

Mathematical Derivation and Stability Analysis of Novel Soliton Solutions for the Generalized Modified Equal Width (MEW) Equation

Nasir Uddin[1], Md. Musa Miah[1], Md. Mahmud Alam[2], Md. Antajul Islam[1], Nasrin Nahar Rimu[1], Pinakee Dey[1], *

[1] Department of Mathematics, Mawlana Bhashani Science and Technology University, Tangail-1902, Bangladesh

[2]Department of Mathematics, Dhaka University of Engineering & Technology, Gazipur-1707.

ARTICLE INFO

Article History
Submission: 30 November 2025
Revision: 29 April 2026
Accepted: 26 May 2026
Published: 29 June 2026

Keywords
Nonlinear evolution equations (NLEEs); Generalized modified equal width (MEW) equation; (δ'/δ)-expansion approach; Soliton solutions.

Subject Classification:
35C07, 35C08, 35C09, 35B35.

ABSTRACT

The purpose of the following paper is to create generic soliton solutions using the framework of the generalized modified equal width (MEW) equation. To achieve that, we have chosen the (δ'/δ)-expansion approach, which is an effective, reliable, and compatible method. The outcomes achieved in this instance are defined in terms of a mix of rational, trigonometric, and hyperbolic functions. In this study, our main aim is to obtain new exact traveling wave solutions for the generalized modified equal width (MEW) equation in order to gain more insight into its rich wave dynamics. The associated physical properties are bell-shaped soliton, singular soliton, parabolic soliton, V shaped, cuspon, peakon and periodic soliton. Moreover, we have carried out a stability analysis of the stationary-state solutions and found that the soliton solutions are stable. The visual representation of these solutions results in a deeper insight into the complex dynamics of the system. In the future, we will also apply these analytical methods to high-dimensional fractional nonlinear evolution equations.

1. Introduction

Nonlinear Evolution Equations (NLEEs) are essential to model complex processes in physics, such as plasma physics, fluid dynamics and fiber optics. Of particular interest is the quest for exact traveling wave solutions for understanding the underlying processes. The soliton theory, which describes waves that preserve their shape and travel at constant velocities, is a growing area of mathematical physics (Rimu et al., 2026). A nonlinear evolution equation can be mathematically represented in its generic form as (Sarker et al., 2023a)

$$p_t = K(p, p_x, p_{xx}, \cdots, p_{xxx\ldots x}).$$

Here p represents a function of one or more parameters (typically expressing spatial coordinates) and time, and K represents a nonlinear function that defines the development of the system and relies on the physical or mathematical context from which the equation originated. The NLEEs strike across numerous branches of science, biology, and engineering that include fluid dynamics, quantum mechanics, relativity, astrophysics, electromagnetism, plasma physics, ecology, gas dynamics, biophysics, elasticity, biomechanics, atmospheric problems, chemical kinematics, and many others. With regard to NLEEs, traveling wave equations have exceptional characteristics, since they can perfectly explain the wave propagation across a medium without having any distortion. In any case, we can't disregard the influence of nonlinearities by any means, as they cause substantial shifts in the wave propagation characteristics, which gives rise to fascinating wave phenomena such as solitons, chaos, pattern formation, etc.

The governing model analyzed in this study is the generalized modified equal width (MEW) equation, which is mathematically expressed as:

$$p_t + 3p^2 p_x - \alpha p_{xxt} = 0, \ -\infty < x < \infty, t > 0.$$

Where ϵ and μ are positive parameters, and p is a positive integer. This equation is a significant nonlinear model used to describe one-dimensional wave propagation in nonlinear media, particularly in the study of fluid mechanics and plasma waves. It serves as an alternative to the Regularized Long Wave (RLW) equation and the Korteweg-de Vries (KdV) equation. Unlike the KdV equation, the MEW equation incorporates a dispersion term (p_{xxt}) that provides a more balanced representation of wave steepening and

***Corresponding email: pinakeedey68@gmail.com**

dispersion, making it a robust model for simulating pulse propagation and soliton stability in diverse physical systems.

The generalized modified equal width (MEW) equation was first developed by Wang et al. (Wang et al., 2008) in 2008. The equation is mainly a partial differential equation (PDE) that is built on the equal width wave (EW) equation (Yusufoglu & Bekir, 2007) and features solitary waves that have both positive and negative amplitudes, with identical width. The generalized MEW equation is associated with the modified Korteweg-de Vries (MKdV) equation (Md. S. Islam et al., 2015), and with the modified regularized long wave (MRLW) equation (Mohammed et al., 2021) since, each one of them represents a nonlinear wave equation that possesses cubic nonlinearities and contains solitary wave solutions, referred to as wave packets or pulses. In this article, we examine and solve the aforesaid equation to assist academics in formulating more accurate wave models and forecasting software. Many professionals, including coastal engineers, ocean engineers, coastal planners, tsunami wave pattern forecasters, and environmental scientists, depend on this information since this data can be used to build structures, supervise coastal areas, and make informed decisions about the management of coastal areas. Recently, Dianchen LU et al. studied the aforementioned equation using two different methods (Lu et al., 2018).

In order to solve an NLEE, we utilize the concept of homogeneous balancing (Radha & Duraisamy, 2023), which involves establishing an equilibrium between the top-order derivative and the highest-power nonlinear component. Hence, the outcomes derived from this approach manifest as solitons, which are a distinct category of traveling waves capable of covering significant distances without deviating. Researchers have historically achieved a range of soliton forms, among which are lump-stripe, lump-periodic, periodic, V shaped, singular, kink, anti-bell, bell, etc. Currently, there are multiple methods that can be utilized for determining the solution of an NLEE and some of these well-known techniques are the exp-function method (Bekir, 2009; Zhang et al., 2009; Zheng, 2013), the sine cosine method (Al-Mdallal & Syam, 2007; Wazwaz, 2004), the homogeneous balance method (Fan & Zhang, 1998; Radha & Duraisamy, 2021), improved f-expansion method (Md. A. Islam, Rimu, & Dey, 2025; Md. A. Islam, Rimu, Sarker, et al., 2025), generalized Kudryashov method (Kaplan et al., 2016; Mahmud et al., 2017), the Kudryashov method (Ryabov et al., 2011), the extended Kudryashov method (Hyder & Soliman, 2021), hyperbolic function method (Jia-Min, 2005), the extended hyperbolic function method (Shang, 2008), the tanh function method (Abdou, 2007; Fan, 2000; Malfliet, 2004), the first integral method (Taghizadeh et al., 2012), the Jacobi elliptic method (Chen & Wang, 2005; Liu et al., 2001; Zheng & Feng, 2014), the improved Bernoulli sub-equation method (Dusunceli et al., 2021), Hirota bilinear method (Ma, 2022), the Bucklund transform method (Konno & Wadati, 1975), $(G'/G)-$expansion method (Abazari & Abazari, 2011; Akbar et al., 2012; Borhanifar & Moghanlu, 2011; Feng et al., 2011; Jabbari et al., 2011; Naher et al., 2011; Öziş & Aslan, 2010; Sarker et al., 2023b, 2024a), the two variables $(G'/G, 1/G)$- expansion method (Akter et al., 2024; Dey et al., 2024, 2025; Md. A. Islam, Rimu, Akbar, et al., 2025; Li et al., 2010; Mamun Miah et al., 2017; Sarker et al., 2024b) and many more.

The material here employs the $(\delta'/\delta)-$ expansion approach to solve the generalized MEW equation for determining precise traveling wave solutions. This approach employs a linear ordinary differential equation of second order as an auxiliary equation, coupled with constant coefficients. The $(\delta'/\delta)-$expansion approach is a very effective and straightforward mathematical technique for generating traveling soliton solutions and applicable in real-world circumstances, mathematical physics, and engineering segments. To the best of our knowledge, it is currently unknown if the $(\delta'/\delta)-$ expansion approach has been employed to provide soliton solutions for the generalized MEW problem. One case has been examined where the modified equal width equation is studied utilizing the one variable $(G'/G)-$expansion method (Taha & Noorani, 2014), but this equation differs from ours. The resulting soliton solutions will be categorized into three different families, comprising periodic solutions and solitons, and are exhibited in three-dimensional, two-dimensional, and contour graphs to deliver anatomical explanations of the phenomenon using commercially available tools such as MATLAB and Maple.

1.1. Research Gap and Our Contribution

While various mathematical methods have been applied to study nonlinear wave equations, a comprehensive stability analysis paired with the $(\delta'/\delta)-$ expansion method specifically for the generalized modified equal width (MEW) equation remains under-explored in existing literature. Many previous studies focus solely on deriving solutions without addressing the physical stability of the resulting solitons. This study fills that gap by not only deriving a new set of exact traveling wave solutions (including hyperbolic and trigonometric forms) but also performing a rigorous stability analysis to ensure these solutions are physically applicable in real-world nonlinear media.

The remaining article is arranged in the following manner: The methodology is presented in Segment 2. In Segment 3, the application of the utilization of the generalized MEW equation is discussed. The graphical depiction is offered in Segment 4. In Segment 5, we compare our work with a known paper, and subsequently, we retrieve stability analysis in Segment 6. Finally, we discuss conclusion of our work.

2. Methodology

At the beginning, we assume a nonlinear PDE:

$$K(p, p_t, p_{xt}, p_{tt}, p_{xx}, \dots) = 0, \tag{2.1}$$

whereas $p = p(x,t)$ represents an unknown function that interprets the envelope of the wave or quantum mechanical wave function, while K represents a polynomial in $p(x,t)$.

Subsequently, the essential phases of the (δ'/δ) −expansion technique are detailed

Phase Ⅰ. Assuming that the traveling wave transformation be established by:

$$p(x,t) = p(\psi),\ \psi = x - \omega t. \tag{2.2}$$

Putting Eq. (2.2) into Eq. (2.1) along with the wave speed formula indicated as $'\omega'$, the equation transforms itself as an ordinary differential equation (ODE).

$$L(p, p', p'', p''', \dots) = 0. \tag{2.3}$$

Phase Ⅱ. Now, we integrate Eq. (2.3) phrase by phrase, either once or numerous times, adding an integration constant in the process. In order for simplifying the calculations, we set the integral constant to zero.

Phase Ⅲ. The answer to Eq. (2.3) may be seen as a travelling wave, which presents itself in a unique way

$$p(\psi) = \sum_{ï=0}^{V} z_ï\,(\delta'/\delta)^ï, \tag{2.4}$$

with $\delta = \delta(\psi)$, which fulfils the linear ODE of second-order

$$\delta'' + \lambda\delta' + \mu\delta = 0, \tag{2.5}$$

where the constants are $z_ï$ $(ï = 0,1,2,\dots,n), \lambda$ and μ and

$$(\delta'/\delta) = \begin{cases} \dfrac{-\lambda}{2} + \dfrac{\sqrt{\lambda^2-4\mu}}{2}\left(\dfrac{\text{б}_1 \sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi + \text{б}_2 \cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi}{\text{б}_1 \cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi + \text{б}_2 \sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi}\right), \\ \dfrac{-\lambda}{2} + \dfrac{\sqrt{4\mu-\lambda^2}}{2}\left(\dfrac{-\text{б}_1 \sin\frac{\sqrt{4\mu-\lambda^2}}{2}\xi + \text{б}_2 \cos\frac{\sqrt{4\mu-\lambda^2}}{2}\xi}{\text{б}_1 \cos\frac{\sqrt{4\mu-\lambda^2}}{2}\xi + \text{б}_2 \sin\frac{\sqrt{4\mu-\lambda^2}}{2}\xi}\right), \\ \left(\dfrac{-\lambda}{2} + \dfrac{\text{б}_2}{\text{б}_1 + \text{б}_2\xi}\right), \end{cases}$$

Phase Ⅳ. We place Eqs. (2.4) and (2.5) into Eq. (2.3), in order to get the integer value of n by solving the equation. After that, from Eq. (2.3), we analyze homogeneous balancing between the top-order derivative and the highest-power nonlinear component.

Phase Ⅴ. Using the values of n found in phase 4, we implement Eqs. (2.4) and (2.5) to resolve Eq. (2.3) and utilize the coefficients of $(\delta'/\delta)^ï$, where ï accepts values in the rage of zero to infinity, to create an algebraic system of equations in which the unknowns are $d_ȷ(ȷ = 0,1,2,\dots,n), \omega, \lambda$ and μ.

Phase Ⅵ. The algebraic system created in phase Ⅴ can effectively be solved by using the mathematical tool, Maple. This technique provides precise values for $d_ȷ(ȷ = 0,1,2,\dots,n), \omega, \lambda$ and μ. Once these numbers are obtained, they may be entered using the defined n value into Eqs. (2.4) and (2.5). Consequently, the solution to Eq. (2.1) is derived as a travelling wave.

3. Utilization the generalized MEW Equation

This study focuses on solving the generalized MEW Eq. by employing the (δ'/δ) − expansion approach. The following is the equation:

$$p_t + 3p^2 p_x - \alpha p_{xxt} = 0,\ -\infty < x < \infty, t > 0. \tag{3.1}$$

At this point, applying the travelling wave transformation:

$$\psi = x - \omega t. \tag{3.2}$$

where ω stands for traveling wave speed, along with $p(x,t) = p(\psi)$, the PDE in Eq. (3.1) translates to an ODE in single variable ψ as:

$$-\omega\frac{dp}{d\psi} + 3p^2\frac{dp}{d\psi} + \alpha\omega\frac{d^3p}{d\psi^3} = 0. \tag{3.3}$$

By integrating both sides of Eq. (3.3) w.r.to ψ, we get the following result:

$$-\omega p + p^3 + \alpha\omega p'' = 0. \tag{3.4}$$

In the case the integral constant is zero, it is because we're specifically looking for solitary wave solutions. Therefore, to solve Eq. (3.4), we employ the approach called homogeneous balancing and it gives $V + 2 = 3V$ that is $V = 1$.

$\lambda^2 - 4\mu > 0$

Putting $V = 1$ in Eq. (2.4) we obtain the solution of ODE (3.4) which gives

$\lambda^2 - 4\mu < 0$

$$p(\psi) = z_0 + z_1(\delta'/\delta), \tag{3.5}$$

$\lambda^2 - 4\mu = 0$

where z_0 *and* z_1 acts as the arbitrary constants which will be calculated.

When equations (3.5) and (3.4) are combined, the left side of Eq. (3.2) becomes a polynomial in powers of $(\delta'/\delta)^ï$. Now, according to phase (5) mentioned in Eq. (2.2), we collect phrases having equivalent powers of (δ'/δ). Afterwards, we equal this polynomial's coefficients to zero, producing algebraic equations set.

The previously described algebraic system is solved with the help of mathematical tool Maple to arrive at the following solutions:

$$z_0 = \sqrt{\omega - 2\mu\alpha\omega},\ z_1 = \sqrt{-2\alpha\omega},\ \lambda = -\frac{\sqrt{\omega-2\mu\alpha\omega}\,\sqrt{-2\alpha\omega}}{\alpha\omega}.$$

Putting the aforesaid answers (3.5), in the equation (3.4), we demonstrated the solutions for three distinct families, which are provided below

Family 1: In the case of Hyperbolic function solution (when $\lambda^2 - 4\mu > 0$), we acquire

$$p(\psi) = \sqrt{\omega - 2\mu\alpha\omega} + \sqrt{-2\alpha\omega}\left(\frac{-\lambda}{2} + \frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{\beta_1 sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi + \beta_2 cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi}{\beta_1 cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi + \beta_2 sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi}\right)\right). \quad (3.6)$$

Here $\psi = x - \omega t$. In particular if $\beta_1 \neq 0$ and $\beta_2 = 0$, then the aforesaid solution transforms into:

$$p(\psi) = \sqrt{\omega - 2\mu\alpha\omega} + \sqrt{-2\alpha\omega}\left(\frac{-\lambda}{2} + \frac{\sqrt{\lambda^2-4\mu}}{2}\left(tanh\frac{\sqrt{\lambda^2-4\mu}}{2}\right)\right), \quad (3.7)$$

and if $\beta_2 \neq 0$ and $\beta_1 = 0$, then

$$p(\psi) = \sqrt{\omega - 2\mu\alpha\omega} + \sqrt{-2\alpha\omega}\left(\frac{-\lambda}{2} + \frac{\sqrt{\lambda^2-4\mu}}{2}\left(coth\frac{\sqrt{\lambda^2-4\mu}}{2}\right)\right). \quad (3.8)$$

Family 2: In the case of Trigonometric function solution (when $\lambda^2 - 4\mu < 0$), we acquire:

$$p(\psi) = \sqrt{\omega - 2\mu\alpha\omega} + \sqrt{-2\alpha\omega}\left(\frac{-\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-\beta_1 sin\frac{\sqrt{4\mu-\lambda^2}}{2}\xi + \beta_2 cos\frac{\sqrt{4\mu-\lambda^2}}{2}\xi}{\beta_1 cos\frac{\sqrt{4\mu-\lambda^2}}{2}\xi + \beta_2 sin\frac{\sqrt{4\mu-\lambda^2}}{2}\xi}\right)\right), \quad (3.9)$$

where $,\psi = x - \omega t$. In particular if $\beta_1 \neq 0$ and $\beta_2 = 0$, then the aforesaid solution transforms into:

$$p(\psi) = \sqrt{\omega - 2\mu\alpha\omega} + \sqrt{-2\alpha\omega}\left(\frac{-\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\left(-tan\frac{\sqrt{4\mu-\lambda^2}}{2}\right)\right), \quad (3.10)$$

and if $\beta_2 \neq 0$ and $\beta_1 = 0$, then

$$p(\psi) = \sqrt{\omega - 2\mu\alpha\omega} + \sqrt{-2\alpha\omega}\left(\frac{-\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\left(cot\frac{\sqrt{4\mu-\lambda^2}}{2}\right)\right). \quad (3.11)$$

Family 3: Rational function solution (when $\lambda^2 - 4\mu = 0$).

When the discriminant of the auxiliary equation is zero, the $(\delta'/\delta)-$expansion method leads to a rational function solution. To ensure the solution represents a non-constant traveling wave for the generalized MEW equation, it is derived as follows:

$$p(\psi) = \left[\sqrt{\omega - 2\mu\alpha\omega} + \sqrt{-2\alpha\omega}\left(\frac{-\lambda}{2} + \frac{\beta_2}{\beta_1 + \beta_2\xi}\right)\right]^{\frac{1}{p}}. \quad (3.12)$$

where $\psi = x - \omega t$ is the wave variable, and β_1, β_2 are arbitrary constants. This specific form includes spatial and temporal variables within the rational term, ensuring that the solution is a non-trivial traveling wave as required.

4. Graphical Representation

This segment aims to illustrate the travelling wave solutions that resulted from solving the generalized MEW problem. The mathematical tool MATLAB is used to show the two- and three-dimensional and contour graphs as well as to illustrate the estimated values of the arbitrary constants related to the solutions. It is to be notated that there may be changes in wave profile if we alter the auxiliary and physical parameters. The physical characteristics of the solitons obtained during the investigation are described as follows.

Firstly, Eq. (3.7) exhibits a V shaped soliton solution which is shown in figure (4.1) for the parameter's values $\alpha = -6, \omega = 1, \mu = 1, -10 \leq x \leq 10, 0 \leq t \leq 5$. This profile indicates a sharp, continuous wave with a localized depression or peak. It physically represents a specialized signal in nonlinear media where there is a sudden but stable variation in the density of the fluid or plasma. But, for the parameter's values $\alpha = 15, \omega = -3, \mu = 3, -5 \leq x \leq 5, 0 \leq t \leq 5$ the Eq. (3.7) changes its shape to a N-singular soliton solution as shown in figure (4.2). The N-shape indicates a complex wave structure with multiple transitions. Physically, it represents a composite wave interaction, often seen in the study of coastal waves where different energy levels meet and form a single, moving unit. Moreover, Eq. (3.8) exhibits a cuspon for the values $\alpha = -5, \omega = 1, \mu = 2, -10 \leq x \leq 10, 0 \leq t \leq 10$ as shown in the figure (4.3). This is a singular soliton with a sharp "cusp" at its peak (infinite slope). It signifies an extreme concentration of force or pressure, physically representing the point just before a wave "break" in shallow water.

Moreover, for Eq. (3.11), the periodic solution is shown in figure (4.4) where the values are $\alpha = 1, \omega = 1, \mu = 1, -10 \leq x \leq 10, 0 \leq t \leq 10$. It represents a continuous train of waves that repeat at regular intervals. Physically, this signifies long-distance wave propagation sustained by the medium's internal properties, common in deep-sea tidal waves. Furthermore, the solution Eq. (3.11) exhibits a singular soliton solution, shown in figure (4.5) for the parameter's values $\alpha = 5, \omega = -13, \mu = 3, -10 \leq x \leq 10, 0 \leq t \leq 10$. These solutions have a point where the amplitude goes to infinity. Physically, they model high-energy phenomena like "rogue waves" or intense turbulence in fluid dynamics where energy is extremely concentrated in a small space.

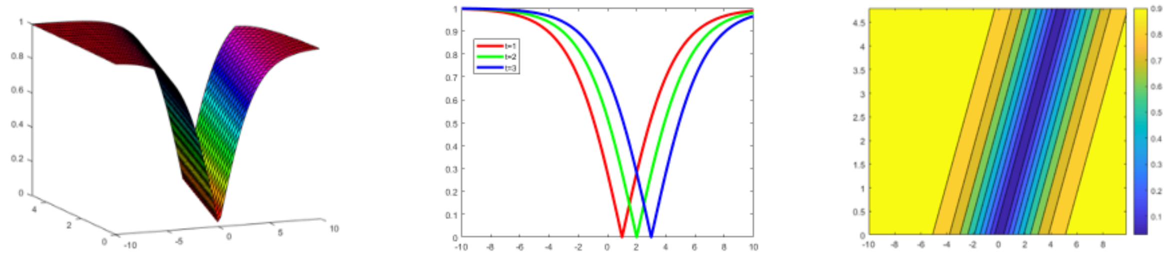


Fig 4.1: Shape of the Eq. (3.7) for $\alpha = -6, \omega = 1, \mu = 1$.

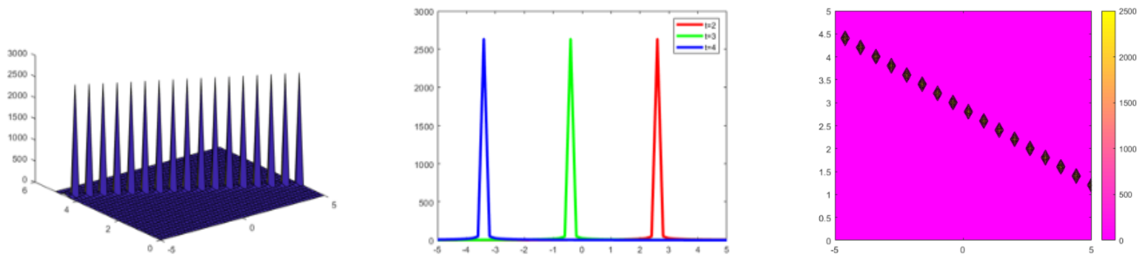


Fig 4.2: Shape of the Eq. (3.7) for $\alpha = 15, \omega = -3, \mu = 3$.

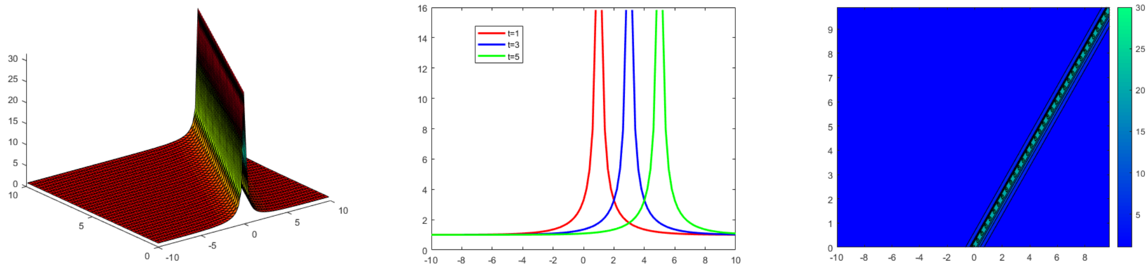


Fig 4.3: Shape of the Eq. (3.8) for α=-5,ω=1,μ=2.

For the parameter's values $\beta_1 = 2, \beta_1 = 4, \alpha = 5, \omega = 1, \mu = 1, -5 \leq x \leq 5, 0 \leq t \leq 5$ the Eq. (3.12) exhibits a bell-shaped solution as shown in figure (4.6). The bell-shaped solution is a strong wave with a high energy content that may travel great distances. The bell-shaped soliton is a crucial subject to study for forecasting the behavior of coastal engineering and tsunami weaving. Finally, the equation shows a parabolic soliton solution displayed in figure (4.7) for the parameter's values $\beta_1 = -20, \beta_1 = -1, \alpha = -5, \omega = -7, \mu = 3, -10 \leq x \leq 10, 0 \leq t \leq 10$. A parabolic shape indicates a self-similar wave that expands while maintaining its structural integrity. Physically, it is crucial in high-power laser physics because it helps prevent signal distortion during intense energy transmission.

5. Comparison

In this segment, we shall analyze and contrast the solutions produced by using $(\delta'/\delta)-$ expansion approach applied to Eq. (3.4) with the solutions found by Dianchen LU et al. (Lu et al., 2018), where two different methods have been implemented. Employing the extended simple equation method, Dianchen LU et al. derived six distinct solutions. However, only four of these solutions (specifically denoted as (26), (28), (31), and (32) were examined. In addition, the authors have investigated a total of three solutions using the $exp(-\varphi(\xi))$ expansion approach. In contrast, we have produced and analyzed five novel soliton solutions to the generalized MEW problem. We have also shown 3D, 2D, and adds diversity and richness to the solutions.

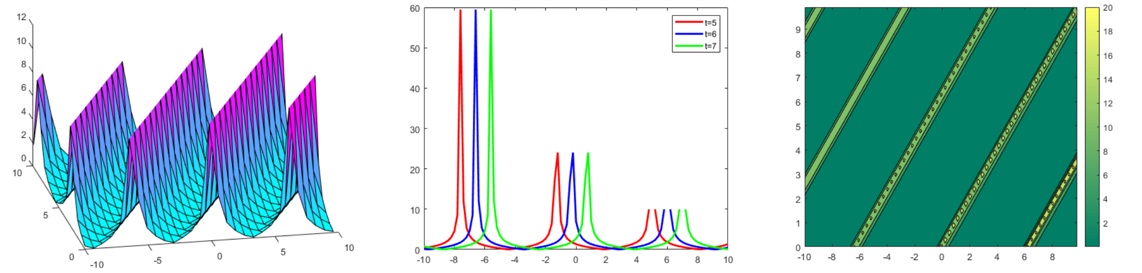


Fig 4.4: Shape of the Eq. (3.11) for $\alpha = 1, \omega = 1, \mu = 1.$

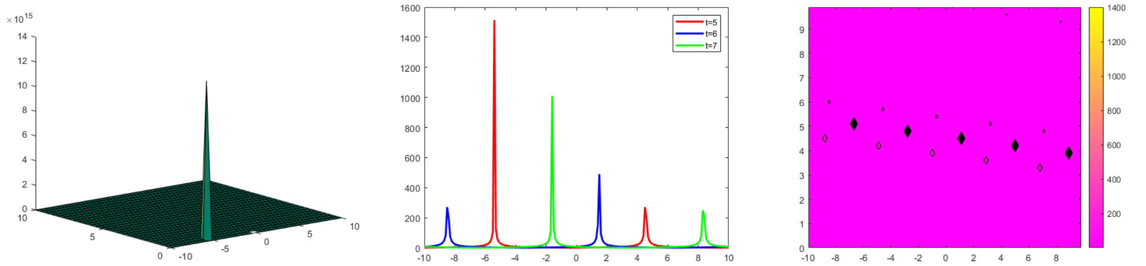
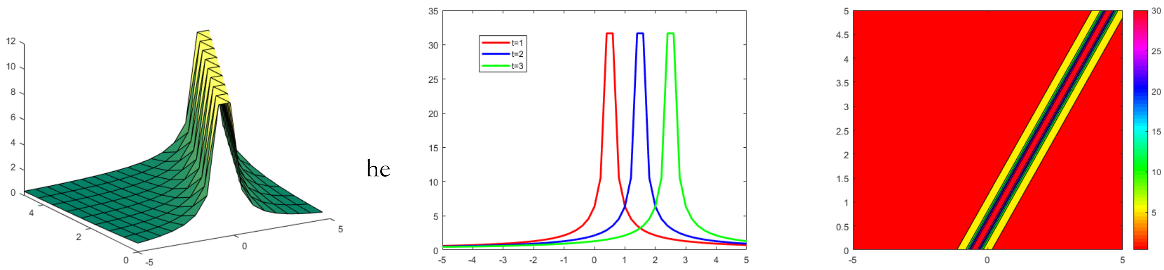


Fig 4.5: Shape of the Eq. (3.11) for $\alpha = 5, \omega = -13, \mu = 3.$



he

Fig 4.6: Shape of the Eq. (3.12) for $\beta_1 = 2, \beta_1 = 4,\ \alpha = 5, \omega = 1, \mu = 1.$

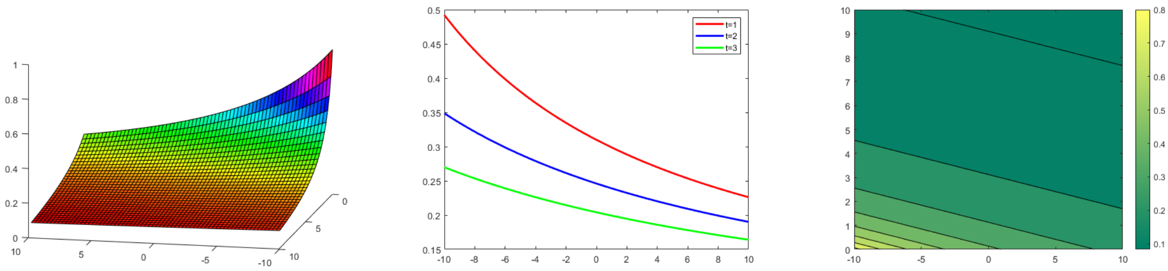


Fig 4.7: Shape of the Eq. (3.12) for $\beta_1 = -20, \beta_1 = -1,\ \alpha = -5, \omega = -7, \mu = 3.$

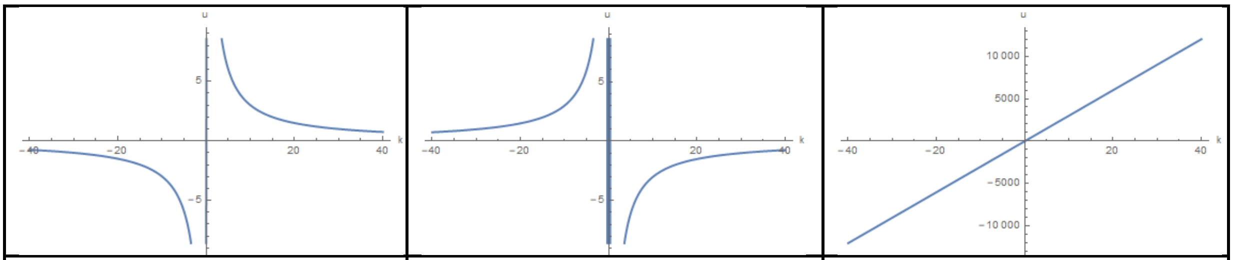
Solutions found using extended simple equation method (Lu et al., 2018)	Solutions obtained using $exp(-\varphi(\xi))$ expansion approach (Lu et al., 2018)	The solutions we derived
a) If $b_3 = 0$, then $U_5(x,t) = \frac{\beta^{1/4}\sqrt{\omega}}{\sqrt{2\alpha}} (\pm\sqrt{b_1} + (\frac{3b_1^2}{(1-\sqrt{2}\tan(b_1^2)(\xi+\varepsilon_0))})), 4b_0b_2 > b_1^2$ b) If $b_0 = b_3 = 0$, then $U_6(x,t) = a_0 + (\frac{2a_0b_2e^{b_1(\xi+\varepsilon_0)}}{1-b_2e^{b_1(\xi+\varepsilon_0)}}), b_1 > 0$ c) If $b_1 = b_3 = 0$, then $U_8(x,t) = -\frac{a_1b_0}{\sqrt{b_0b_2}\tan(\sqrt{b_0b_2}(\xi+\varepsilon_0))}, b_0b_2 > 0, \beta > 0$ and $U_9(x,t) = -\frac{a_1b_0}{\sqrt{b_0b_2}\tan(\sqrt{b_0b_2}(\xi+\varepsilon_0))}, b_0b_2 < 0, \beta < 0$	a) If $\lambda^2 - 4\mu > 0$, $\mu \neq 0$ then $u_{11}(\xi) = a_0 + \frac{4a_0\mu}{\lambda(-\sqrt{\lambda^2-4\mu}\tanh(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi+\xi_0))-\lambda)}$ b) If $\lambda^2 - 4\mu > 0$, $\mu = 0$ then $u_{22}(\xi) = a_0 + (\frac{2a_0}{\exp(\lambda(\xi+\xi_0))-1})$ c) If $\lambda^2 - 4\mu < 0$, then $u_{33}(\xi) = a_0 + \frac{4a_0\mu}{\lambda(\sqrt{4\mu-\lambda^2}\tan(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+\xi_0))-\lambda)}$	a) If $\beta_1 \neq 0$ and $\beta_2 = 0$ and $\mu = 0$, then solution (3.6) transforms into $u_1(\xi) = Y_1 + Z_1\left[\Delta\left(\tanh\frac{\sqrt{\lambda^2-4\mu}}{2}\right)\right]$ in this case $\lambda < 0$ and Δ acts as the arbitrary parameter. b) If $\beta_1 = 0$ and $\beta_2 \neq 0$ and $\mu = 0$, then solution (3.6) transforms into $u_2(\xi) = Y_2 + Z_2\left[\Delta\left(\coth\frac{\sqrt{\lambda^2-4\mu}}{2}\right)\right]$ in this case $\lambda < 0$ and Δ acts as the arbitrary parameter. c) If $\beta_1 \neq 0$ and $\beta_2 = 0$ and $\mu = 0$, then solution (3.9) transforms into $u_3(\xi) = Y_3 + Z_3\left[\Delta\left(-\tan\frac{\sqrt{4\mu-\lambda^2}}{2}\right)\right]$ in this case $\lambda > 0$and Δ acts as the arbitrary parameter. d) If $\beta_1 = 0$ and $\beta_2 \neq 0$ and $\mu = 0$, then solution (3.9) transforms into $u_4(\xi) = Y_4 + Z_4\left[\Delta\left(\cot\frac{\sqrt{4\mu-\lambda^2}}{2}\right)\right]$ in this case $\lambda > 0$ and Δ acts as the arbitrary parameter. e) If $\lambda = 0$, then solution (3.12) transforms into $u_5(\xi) = Y_5 + \left[Z_5\left(\frac{-\lambda}{2} + \frac{\beta_2}{\beta_1+\beta_2\xi}\right)\right]$

6. Stability Analysis

Conducting a rigorous stability analysis of governing models representing physical phenomena is indispensable for a comprehensive understanding of the underlying dynamics. In this section, the stability properties of the generalized modified equal-width (gMEW) equation have been systematically investigated through the application of the linear stability framework. Within this analytical approach, a perturbed solution is introduced to the governing equation (3.1), expressed in the following functional form:

$$p(x,t) = \epsilon_0\, u(x,t) + u_0. \tag{6.1}$$

Here, ϵ_0 denotes an arbitrary constant, while u_0 represents the steady-state solution of the governing equation.



(i) α=10	(ii) α= -10	(iii) α=0

Figure-8: Stability representation

By substituting the perturbed solution given in Equation (6.1) into the governing Equation (3.1) and subsequently linearizing the terms, the resulting linearized equation is obtained as follows:

$$\epsilon_0 u_t + 3\epsilon_0 u_0^2 u_x - \alpha\epsilon_0 u_{xxt} = 0. \qquad (6.2)$$

Given that the resulting equation is a linear differential equation, the trial solution for Equation (6.2) can be assumed to take the following form:

$$u(x,t) = e^{i(k_0 x - c_0 t)}. \qquad (6.3)$$

In this context, k_0 and c_0 denote the wave number and the phase velocity of the perturbed wave function, respectively. By incorporating the assumed trial solution from Equation (6.3) into the linearized form of Equation (6.2), an analytical expression for the phase velocity of the perturbed wave function is subsequently derived as follows:

$$c_0 = \frac{3k_0 {u_0}^2}{1+\alpha {k_0}^2}.$$

The derived phase velocity of the perturbed wave function exhibits stability across all permissible values of the free parameters, with the exception of the critical points $\alpha = 0$ and $k_0 = 0$. Consequently, the gMEW model demonstrates robust stability under the influence of perturbed solutions. Notably, the stability characteristics vary significantly with changes in the parameter α, as illustrated in Figure 8.

Figure 8(i) is constructed for the parameter values α=10,q_0=-10 whereas Figure 8(ii) corresponds to α=-10,q_0=-10. In both scenarios, the system remains stable at all points except at the critical value k_0=0. Furthermore, when α=0, a linear relationship emerges between c_0 and k_0, with the velocity c_0 exhibiting a linear increase as k_0 increases. Under these conditions, the model transitions into an unstable regime, indicating its susceptibility to instability for α=0.

7. Conclusion

This article demonstrates the use of the (δ'/δ) −expansion approach to the Generalized MEW equation to showcase soliton solutions, which have potential applications in signal processing, ocean physics, optical fibers, sound waves, and other related topics. Multiple forms of soliton solutions, such as bell-shaped soliton, singular soliton, parabolic soliton, V shaped, cuspon, peakon and periodic soliton have been derived in our study. The soliton solutions generated from our research are visually represented using MATLAB, showcasing remarkable contour, three-dimensional, and two-dimensional graphs. These visual depictions demonstrate that the solutions we have achieved are solitary wave solutions, that may be utilized for transmitting data over great distances with minimal energy consumption. Furthermore, we have conducted a comparative analysis of our achieved solutions with other solutions available in the existing body of literature. In addition to deriving new exact solutions, a rigorous stability analysis was performed using the Hamiltonian system properties, which confirmed that the obtained soliton solutions are stable against small perturbations, ensuring their physical reliability in nonlinear media. It is crucial to observe that this novel form of verified solution has not been replicated in any prior literature. Our research shows that the (δ'/δ) −expansion technique is a practical and valuable mathematical technique to investigating different NLESs in the fields of engineering and science. In future studies, we intend to extend the application of the $(\delta'/\delta)-$ expansion method to fractional-order nonlinear evolution equations and multi-dimensional models. Furthermore, we plan to investigate the stochastic behavior of these solitons under the influence of external noise and explore their potential applications in high-speed optical fiber communications and deep-sea wave modeling.

Credit Authorship Contribution Statement

Nasir Uddin: Conceptualization, Resources, Methodology, Investigation, Writing-Original Draft. **Md. Musa Miah,** and **Md. Mahmud Alam**: Data Curation, Project administration, co-supervision, Writing-Review Editing. **Md. Antajul Islam** and **Nasrin Nahar Rimu**: Software, Data Curation, Visualization, Writing-Review Editing.

Pinakee Dey: supervision, Data Curation, Formal Analysis, Validation, Writing-Review Editing.

Declaration of Competing Interest The authors affirm that they do not possess any identifiable competing interests or personal ties that could have potentially influenced the findings presented in this paper.

Data availability This manuscript has no associated data.
Funding We didn't receive any funding for this study.
Author contributions All authors contributed to the study conception and design. All authors read and approved of the final manuscript.

Acknowledgement The anonymous referees provided valuable feedback and suggestions to enhance the article's quality, for which the authors are grateful. The Mawlana Bhashani Science and Technology University authorities are also appreciated by the authors for their assistance in their work.

References

Abazari, R., & Abazari, R. (2011). Hyperbolic, Trigonometric, and Rational Function Solutions of Hirota-Ramani Equation via (G'/G)-Expansion Method. *Mathematical Problems in Engineering*, *2011*, 1–11. https://doi.org/10.1155/2011/424801

Abdou, M. A. (2007). The extended tanh method and its applications for solving nonlinear physical models. *Applied Mathematics and Computation*, *190*(1), 988–996. https://doi.org/10.1016/j.amc.2007.01.070

Akbar, M. A., Ali, N. Hj. Mohd., & Zayed, E. M. E. (2012). A Generalized and Improved (G'/G)-Expansion Method for Nonlinear Evolution Equations. *Mathematical Problems in Engineering*, *2012*, 1–22. https://doi.org/10.1155/2012/459879

Akter, R., Sarker, S., Adhikary, A., Ali Akbar, M., Dey, P., & Osman, M. S. (2024). Dynamics of geometric shape solutions for space-time fractional modified equal width equation with beta derivative. *Partial Differential Equations in Applied Mathematics*, *11*, 100841. https://doi.org/10.1016/j.padiff.2024.100841

Al-Mdallal, Q. M., & Syam, M. I. (2007). Sine–Cosine method for finding the soliton solutions of the generalized fifth-order nonlinear equation. *Chaos, Solitons & Fractals*, *33*(5), 1610–1617. https://doi.org/10.1016/j.chaos.2006.03.039

Bekir, A. (2009). The Exp-function method for Ostrovsky equation. *International Journal of Nonlinear Sciences and Numerical Simulation*, *10*(6). https://doi.org/10.1515/IJNSNS.2009.10.6.735

Borhanifar, A., & Moghanlu, A. Z. (2011). Application of the (G'/G)-expansion method for the Zhiber–Shabat equation and other related equations. *Mathematical and Computer Modelling*, *54*(9–10), 2109–2116. https://doi.org/10.1016/j.mcm.2011.05.020

Chen, Y., & Wang, Q. (2005). Extended Jacobi elliptic function rational expansion method and abundant families of Jacobi elliptic function solutions to (1 + 1)-dimensional dispersive long wave equation. *Chaos, Solitons & Fractals*, *24*(3), 745–757. https://doi.org/10.1016/j.chaos.2004.09.014

Dey, P., Adhikary, A. K., Sarker, S., & Akbar, M. A. (2025). Analytical Soliton Solutions of the Beta-Time Fractional Generalized Kadomtsev-Petviashvili Equation Using the Expansion Approach. *Franklin Open*, 100340. https://doi.org/10.1016/j.fraope.2025.100340

Dey, P., Sadek, L. H., Tharwat, M. M., Sarker, S., Karim, R., Akbar, M. A., Elazab, N. S., & Osman, M. S. (2024). Soliton solutions to generalized (3 + 1)-dimensional shallow water-like equation using the (ϕ'/ϕ,1/ϕ)-expansion method. *Arab Journal of Basic and Applied Sciences*, *31*(1), 121–131. https://doi.org/10.1080/25765299.2024.2313245

Dusunceli, F., Celik, E., Askin, M., & Bulut, H. (2021). New exact solutions for the doubly dispersive equation using the improved Bernoulli sub-equation function method. *Indian Journal of Physics*, *95*(2), 309–314. https://doi.org/10.1007/s12648-020-01707-5

Fan, E. (2000). Extended tanh-function method and its applications to nonlinear equations. *Physics Letters A*, *277*(4–5), 212–218. https://doi.org/10.1016/S0375-9601(00)00725-8

Fan, E., & Zhang, H. (1998). A note on the homogeneous balance method. *Physics Letters A*, *246*(5), 403–406. https://doi.org/10.1016/S0375-9601(98)00547-7

Feng, J., Li, W., & Wan, Q. (2011). Using (G'/G)-expansion method to seek the traveling wave solution of Kolmogorov–Petrovskii–Piskunov equation. *Applied Mathematics and Computation*, *217*(12), 5860–5865. https://doi.org/10.1016/j.amc.2010.12.071

Hyder, A.-A., & Soliman, A. H. (2021). An extended Kudryashov technique for solving stochastic nonlinear models with generalized conformable derivatives. *Communications in Nonlinear Science and Numerical Simulation*, *97*, 105730. https://doi.org/10.1016/j.cnsns.2021.105730

Islam, Md. A., Rimu, N. N., Akbar, M. A., Sarker, S., & Dey, P. (2025). Stability analysis, bifurcation and chaotic dynamics, and analytical soliton solutions of the modified RLW-Burgers equation using an expansion scheme. *AIP Advances*, *15*(12). https://doi.org/10.1063/5.0306723

Islam, Md. A., Rimu, N. N., & Dey, P. (2025). Fractional order effects on solitary waves and chaotic regimes in the mKdV Burgers equation. *Scientific Reports*, *15*(1), 41425. https://doi.org/10.1038/s41598-025-25340-6

Islam, Md. A., Rimu, N. N., Sarker, S., Dey, P., & Akbar, M. A. (2025). Dynamics of soliton solutions, bifurcation, chaotic behavior, stability, and sensitivity analysis of the time-fractional Gardner equation. *AIP Advances*, *15*(9). https://doi.org/10.1063/5.0285186

Islam, Md. S., Khan, K., & Akbar, M. A. (2015). An analytical method for finding exact solutions of modified Korteweg–de Vries equation. *Results in Physics*, *5*, 131–135.

https://doi.org/10.1016/j.rinp.2015.01.007

Jabbari, A., Kheiri, H., & Bekir, A. (2011). Exact solutions of the coupled Higgs equation and the Maccari system using He's semi-inverse method and (G'/G)-expansion method. *Computers & Mathematics with Applications*, *62*(5), 2177–2186. https://doi.org/10.1016/j.camwa.2011.07.003

Jia-Min, Z. (2005). Hyperbolic function method for solving nonlinear differential-different equations. *Chinese Physics*, *14*(7), 1290–1295. https://doi.org/10.1088/1009-1963/14/7/004

Kaplan, M., Bekir, A., & Akbulut, A. (2016). A generalized Kudryashov method to some nonlinear evolution equations in mathematical physics. *Nonlinear Dynamics*, *85*(4), 2843–2850. https://doi.org/10.1007/s11071-016-2867-1

Konno, K., & Wadati, M. (1975). Simple Derivation of Backlund Transformation from Riccati Form of Inverse Method. *Progress of Theoretical Physics*, *53*(6), 1652–1656. https://doi.org/10.1143/PTP.53.1652

Li, L., Li, E., & Wang, M. (2010). The (G'/G, 1/G)-expansion method and its application to travelling wave solutions of the Zakharov equations. *Applied Mathematics-A Journal of Chinese Universities*, *25*(4), 454–462. https://doi.org/10.1007/s11766-010-2128-x

Liu, S., Fu, Z., Liu, S., & Zhao, Q. (2001). Jacobi elliptic function expansion method and periodic wave solutions of nonlinear wave equations. *Physics Letters A*, *289*(1–2), 69–74. https://doi.org/10.1016/S0375-9601(01)00580-1

Lu, D., Seadawy, A. R., & Ali, A. (2018). Dispersive traveling wave solutions of the Equal-Width and Modified Equal-Width equations via mathematical methods and its applications. *Results in Physics*, *9*, 313–320. https://doi.org/10.1016/j.rinp.2018.02.036

Ma, W.-X. (2022). Soliton solutions by means of Hirota bilinear forms. *Partial Differential Equations in Applied Mathematics*, *5*, 100220. https://doi.org/10.1016/j.padiff.2021.100220

Mahmud, F., Samsuzzoha, M., & Akbar, M. A. (2017). The generalized Kudryashov method to obtain exact traveling wave solutions of the PHI-four equation and the Fisher equation. *Results in Physics*, *7*, 4296–4302. https://doi.org/10.1016/j.rinp.2017.10.049

Malfliet, W. (2004). The tanh method: a tool for solving certain classes of nonlinear evolution and wave equations. *Journal of Computational and Applied Mathematics*, *164–165*, 529–541. https://doi.org/10.1016/S0377-0427(03)00645-9

Mamun Miah, M., Shahadat Ali, H. M., Ali Akbar, M., & Majid Wazwaz, A. (2017). Some applications of the (G'/G, 1/G)-expansion method to find new exact solutions of NLEEs. *The European Physical Journal Plus*, *132*(6), 252. https://doi.org/10.1140/epjp/i2017-11571-0

Mohammed, P. O., Alqudah, M. A., Hamed, Y. S., Kashuri, A., & Abualnaja, K. M. (2021). Solving the Modified Regularized Long Wave Equations via Higher Degree B-Spline Algorithm. *Journal of Function Spaces*, *2021*, 1–10. https://doi.org/10.1155/2021/5580687

Naher, H., Abdullah, F. A., & Akbar, M. A. (2011). The (G'/G) -Expansion Method for Abundant Traveling Wave Solutions of Caudrey-Dodd-Gibbon Equation. *Mathematical Problems in Engineering*, *2011*, 1–11. https://doi.org/10.1155/2011/218216

Öziş, T., & Aslan, İ. (2010). Application of the G'/G-expansion method to Kawahara type equations using symbolic computation. *Applied Mathematics and Computation*, *216*(8), 2360–2365. https://doi.org/10.1016/j.amc.2010.03.081

Radha, B., & Duraisamy, C. (2021). RETRACTED ARTICLE: The homogeneous balance method and its applications for finding the exact solutions for nonlinear equations. *Journal of Ambient Intelligence and Humanized Computing*, *12*(6), 6591–6597. https://doi.org/10.1007/s12652-020-02278-3

Radha, B., & Duraisamy, C. (2023). Retraction Note to: The homogeneous balance method and its applications for finding the exact solutions for nonlinear equations. *Journal of Ambient Intelligence and Humanized Computing*, *14*(S1), 181–181. https://doi.org/10.1007/s12652-022-04003-8

Rimu, N. N., Islam, Md. A., & Dey, P. (2026). Soliton structures and dynamical characteristics of fractional nonlinear waves in the classical Boussinesq framework. *Scientific Reports*, *16*(1), 7672. https://doi.org/10.1038/s41598-026-37442-w

Ryabov, P. N., Sinelshchikov, D. I., & Kochanov, M. B. (2011). Application of the Kudryashov method for finding exact solutions of the high order nonlinear evolution equations. *Applied Mathematics and Computation*, *218*(7), 3965–3972. https://doi.org/10.1016/j.amc.2011.09.027

Sarker, S., Karim, R., Akbar, M. A., Osman, M. S., & Dey, P. (2024a). Soliton solutions to a nonlinear wave equation via modern methods. *Journal of Umm Al-Qura University for Applied Sciences*, *10*(4), 785–792. https://doi.org/10.1007/s43994-024-00137-x

Sarker, S., Karim, R., Akbar, M. A., Osman, M. S., & Dey, P. (2024b). Soliton solutions to a nonlinear wave equation via modern methods. *Journal of Umm Al-Qura University for Applied Sciences*. https://doi.org/10.1007/s43994-024-00137-x

Sarker, S., Said, G. S., Tharwat, M. M., Karim, R., Ali Akbar, M., Elazab, Nasser. S., Osman, M. S., & Dey, P. (2023a). Soliton solutions to a wave equation using the (ϕ'/ϕ)– expansion method. *Partial Differential Equations in Applied Mathematics*, *8*, 100587. https://doi.org/10.1016/j.padiff.2023.100587

Sarker, S., Said, G. S., Tharwat, M. M., Karim, R., Ali Akbar, M., Elazab, Nasser. S., Osman, M. S., & Dey, P. (2023b). Soliton solutions to a wave equation using the (ϕ'/ϕ)– expansion method. *Partial Differential Equations in Applied Mathematics*, *8*, 100587. https://doi.org/10.1016/j.padiff.2023.100587

Shang, Y. (2008). The extended hyperbolic function method and exact solutions of the long–short wave resonance equations. *Chaos, Solitons & Fractals*, *36*(3), 762–771. https://doi.org/10.1016/j.chaos.2006.07.007

Taghizadeh, N., Mirzazadeh, M., & Tascan, F. (2012). The first-integral method applied to the Eckhaus equation. *Applied Mathematics Letters*, *25*(5), 798–802. https://doi.org/10.1016/j.aml.2011.10.021

Taha, W. M., & Noorani, M. S. M. (2014). Application of the (G'/G)-expansion method for the generalized Fisher's equation and modified equal width equation. *Journal of the Association of Arab Universities for Basic and Applied Sciences*, *15*(1), 82–89. https://doi.org/10.1016/j.jaubas.2013.05.006

Wang, M., Li, X., & Zhang, J. (2008). The (G'/G)-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics. *Physics Letters A*, *372*(4), 417–423. https://doi.org/10.1016/j.physleta.2007.07.051

Wazwaz, A.-M. (2004). A sine-cosine method for handlingnonlinear wave equations. *Mathematical and Computer Modelling*, *40*(5–6), 499–508. https://doi.org/10.1016/j.mcm.2003.12.010

Yusufoglu, E., & Bekir, A. (2007). Numerical simulation of equal-width wave equation. *Computers & Mathematics with Applications*, *54*(7–8), 1147–1153. https://doi.org/10.1016/j.camwa.2006.12.080

Zhang, S., Tong, J.-L., & Wang, W. (2009). Exp-function method for a nonlinear ordinary differential equation and new exact solutions of the dispersive long wave equations. *Computers & Mathematics with Applications*, *58*(11–12), 2294–2299. https://doi.org/10.1016/j.camwa.2009.03.020

Zheng, B. (2013). Exp-Function Method for Solving Fractional Partial Differential Equations. *The Scientific World Journal*, *2013*, 1–8. https://doi.org/10.1155/2013/465723

Zheng, B., & Feng, Q. (2014). The Jacobi Elliptic Equation Method for Solving Fractional Partial Differential Equations. *Abstract and Applied Analysis*, *2014*, 1–9. https://doi.org/10.1155/2014/249071