Journal of Science and Technology, 5(1): 75-84, January 2015

HEAT GENERATION EFFECT ON CONVECTIVE HEAT TRANSFER FOR CYLINDER WITH RADIATIVE HEAT TRANSFER

ROWSANARA AKHTER[1*], MOHAMMAD MOKADDES ALI[2], M. SHARIF UDDIN[1] and A. R. KHAN[1]

[1]Department of Mathematics, Jahangirnagar University, Savar, Dhaka-1342, Bangladesh
[2]Department of Mathematics, Mawlana Bhashani Science and Technology University, Tangail-1902, Bangladesh

Abstract

In this paper, a steady two-dimensional free convection boundary layer flow and heat transfer of a viscous and incompressible fluid about a circular cylinder in presence of thermal radiation and heat generation is considered. The partial differential equations, governing the problem have been converted employing a set of suitable transformations in a system of non-linear partial differential equations which is solved by using an implicit finite difference method. Numerical calculations are carried out for various values of radiation parameter and heat generation parameter and then presented graphically. It is worth pointing out that, increasing radiation and heat generation leads to enhance the velocity and temperature of the fluid. The results are found to be in good agreement with the existing results.

Keywords: Heat generation, radiation, finite difference method, circular cylinder

Introduction

The problem of convective heat transfer because of circular cylinder driven by buoyancy forces in a viscous incompressible fluid is of great importance in numerous engineering applications. In addition, the radiative heat transfer plays a significant role on the heat transfer systems which may change the flow behavior of various fluids. Hossain *et al.* (1998) examined the effect of conduction-radiation on natural convection flow of an optically dense viscous incompressible fluid over cylinders of elliptic cross section. Chamkha and Quadri (2001) used an implicit, iterative, finite-difference method to solve the problem of coupled heat and mass transfer effect on natural convection for a horizontal cylinder embedded in a uniform porous medium in the presence of an external magnetic field and internal heat generation or absorption. Hossain *et al.* (2001) obtained the series solution and asymptotic solutions of the effect of radiation on free convection flow for a porous vertical plate with variable viscosity. The effects of radiation on magneto-hydrodynamic unsteady free convection flow over a heated vertical plate were analyzed by El-Naby *et al.* (2003). Molla *et al.* (2005) employed implicit finite difference method together with Keller box scheme to investigate the effect of temperature

* Corresponding author: rakhter309@gmail.com

dependent viscosity on free convection flow over an isothermal horizontal circular cylinder. Cheng (2006) applied the cubic spline collocation method to study the natural convection heat and mass transfer in a micropolar fluid for a horizontal cylinder of elliptic cross section with constant wall temperature and concentration. The velocity and temperature distributions of the natural convection flow due to the effects of magnetic field, viscous dissipation and heat generation were studied by Mamun *et al.* (2008) using implicit finite difference techniques. Mukhopadhyay (2009) obtained a numerical solution for the effects of radiation on the boundary layer flow and heat transfer of a fluid with variable viscosity along a symmetric wedge by considering the Lie group transformations. A parametric study of the physical parameters for the effect of radiation and thermal conductivity on MHD free convection flow along a vertical plate had been examined by Akhter *et al.* (2013) using implicit finite difference method together with Keller box techniques.

Motivated by the above studies, the effects of radiative heat transfer from a heated circular cylinder in a viscous incompressible fluid with internal heat generation in ongoing research are analyzed. The governing equations are transformed into non-dimensional form employing appropriate transformations and then solved numerically by the implicit finite difference method together with Keller box scheme (Keller, 1978; Cebeci and Bradshow, 1984). The results are presented graphically and in tabular form and also compared with the previously accomplished results.

Nomenclature

b	thickness of the cylinder	T_f	Temperature of the fluid
C_{fx}	Local skin friction coefficient	T_∞	Temperature of the ambient fluid
c_p	Specific heat at constant pressure	$\bar{u}$, $\bar{v}$	Velocity components
f	Dimensionless stream function	u , v	Dimensionless velocity components
g	Acceleration due to gravity	$\bar{x}$, $\bar{y}$	Cartesian co-ordinates
Gr	Grashof number	x , y	Dimensionless Cartesian co-ordinate
k_f, k_s	Fluid and solid thermal conductivities	β	Coefficient of thermal expansion
Nu_x	Nusselt number	η	Dimensionless similarity variable
p	Conjugate conduction parameter	θ	Dimensionless temperature
Pr	Prandtl number	μ	Viscosity of the fluid
q_w	Heat flux	ν	Kinematic viscosity
Q	Heat generation parameter	ρ	Density of the fluid
Ra	Radiation parameter	τ_w	Shearing stress
R	Radius of the cylinder	ψ	Steam function
T_b	Temperature at inner surface of the cylinder		

Mathematical modeling

We consider a steady free convection boundary layer flow over a circular cylinder in a viscous, incompressible and electrically conducting fluid of low Prandtl number (Pr = 0.70). Here R is the radius with respect to the outer surface and b is the thickness of the cylinder. The inner surface of the cylinder is held at a constant temperature T_b which is greater than the ambient temperature T_∞ and the surface of the cylinder is held at a constant heat flux q_w. The $\bar{x}$ axis is taken along the circumference of the cylinder which is measured from the lower stagnation point and $\bar{y}$ axis is considered normal to the surface. The physical configuration and the coordinates system are shown in Fig. 1.

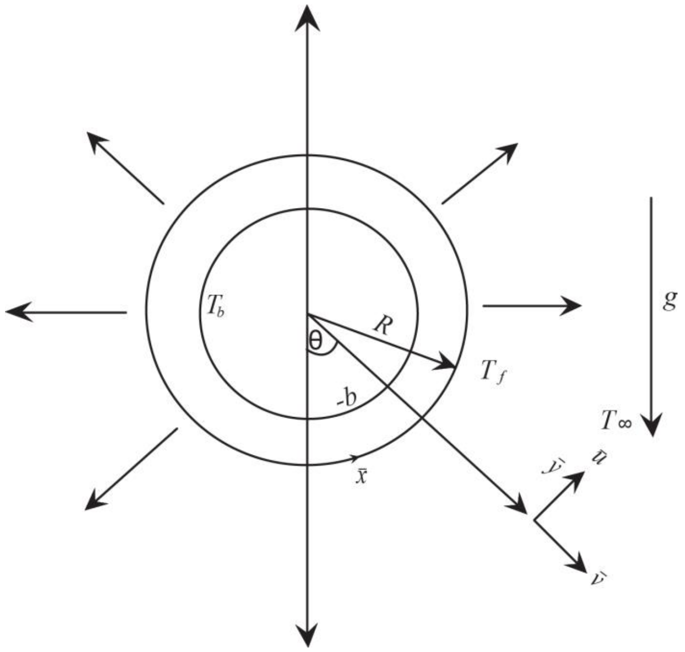

Fig. 1. Physical model and coordinate system

With the usual boundary layer and Bousinesq approximation, the governing equations for free convection flow with internal heat generation and thermal readiation designed for a circular cylinder can be expressed within the typical boundary layer as :

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0\,, \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \nu\frac{\partial^2 \bar{u}}{\partial \bar{y}} + g\beta(T_f - T_\infty)\sin\frac{\bar{x}}{R}\,, \tag{2}$$

$$\bar{u}\frac{\partial T_f}{\partial \bar{x}} + \bar{v}\frac{\partial T_f}{\partial \bar{y}} = \frac{1}{\rho\, c_p}\left(k_f\frac{\partial^2 T_f}{\partial \bar{y}^2} + Q_0\left(T_f - T_\infty\right)\right) - 4\Gamma\left(T_f - T_b\right)\,, \tag{3}$$

Where $\Gamma = \int_0^\infty K_{\lambda b}\left(\frac{\partial e_{b\lambda}}{\partial T_f}\right)_b d\lambda$, $K_{\lambda b} = K_\lambda(T_b)$ is the mean absorption coefficient, $e_{b\lambda}$ is Plank's function and T_f is the temperature of the fluid in the boundary layer, $\nu = \frac{\mu}{\rho}$ is the kinematicviscosity, β is the thermal expression co-efficient, , c_p *is* the specific heat due to constant pressure. The boundary conditions are:

$$\left.\begin{aligned} &\bar{u} = \bar{v} = 0, \quad T_f = T(\bar{x},0), \quad \frac{\partial T_f}{\partial \bar{y}} = \frac{k_s}{b k_f}\left(T_f - T_b\right) \; at \; \bar{y}=0, \; \bar{x}>0 \\ &\bar{u} \to 0, \; T_f \to T_\infty \quad at \;\; \bar{y} \to \infty, \bar{x} > 0 . \end{aligned}\right\} \tag{4}$$

To carry on this analysis, the following dimensionless variables are introduced

$$\left. x = \frac{\bar{x}}{R}, \; y = \frac{\bar{y}}{R} Gr^{1/2}, \; u = \frac{\bar{u} R}{\nu} Gr^{-1/2}, \; v = \frac{\bar{v} R}{\nu} Gr^{-1/4}, \theta = \frac{T_f - T_\infty}{T_b - T_\infty}, \; Gr = \frac{g \beta R^3 (T_b - T_\infty)}{\nu^2} \right\}, \tag{5}$$

where *Gr* is the Grashof number and θ is the non-dimensional temperature. Using these new variables (Equation (5)), the governing equations (1) – (3) become:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \;, \tag{6}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \theta \sin x \;, \tag{7}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} - Ra(\theta - 1) + Q\theta \;. \tag{8}$$

The boundary conditions (4) then reduce to the following form:

$$\left.\begin{aligned} &u = v = 0, \; \theta - 1 = p\frac{\partial \theta}{\partial y} \quad at \quad y = 0, x > 0 \\ &u \to 0, \; \theta \to 0 \qquad\qquad at \quad y \to \infty, x > 0 \end{aligned}\right\} . \tag{9}$$

Here $Ra = \frac{4\Gamma R^2}{\nu} Gr^{-1/2}$ is the radiation parameter, $Pr = \frac{\mu c_p}{k_f}$ is the Prandtl number, $Q = \frac{Q_0 R^2}{\nu \rho} Gr^{-\frac{1}{2}}$ is the heat generation parameter and $p = \left(\frac{k_f}{k_s}\right)\left(\frac{b}{R}\right) Gr^{1/4}$ is a conjugate conduction parameter. In the present analysis we have taken $p = 1$.

To facilitate the solution of this problem, the steam function and the dimensionless temperature are considered as

$$\psi = x f(x,y) \text{ and } \theta = \theta(x,y). \tag{10}$$

The stream function ψ, satisfies the continuity equation (1) through $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$. Employing equation (10) in equations (7) and (8), the governing momentum and energy equations take the simplified form as:

$$f''' + ff'' - f'^2 + \theta \frac{\sin x}{x} = x\left(f' \frac{\partial f'}{\partial x} - f'' \frac{\partial f}{\partial x} \right), \tag{11}$$

$$\frac{1}{\Pr}\theta'' + f\theta' - Ra(\theta - 1) + Q\theta = x\left(f' \frac{\partial \theta}{\partial x} - \theta' \frac{\partial f}{\partial x} \right). \tag{12}$$

The boundary condition (9) become

$$\left.\begin{array}{l} f(x,0) = f'(x,0) = 0, \quad \theta(x,0) = \frac{\partial \theta}{\partial y} \; at \; y = 0 \\ f'(x,\infty) \to 0, \quad \theta(x,\infty) \to 0 \quad as \; y \to \infty \end{array}\right\}. \tag{13}$$

In the above equation, the primes denote partial differentiation with respect to y. The set of equation (11) and (12) together with the boundary condition (13) are solved numerically by applying implicit finite difference method with Keller-box scheme (Keller, 1978; Cebeci and Bradshow, 1984). In practical applications, the physical quantities, *e.g,* local skin friction factor and the local heat transfer are calculated. These quantities are written as follows:

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U_\infty^2} \text{ and } Nu = \frac{\bar{x}\, q_w}{k_f\,(T_b - T_\infty)}, \tag{14}$$

where $\tau_w = \mu \left(\frac{\partial \bar{u}}{\partial \bar{y}} \right)_{\bar{y}=0}$ and $q_w = -k_f \left(\frac{\partial T_f}{\partial \bar{y}} \right)_{\bar{y}=0}$ are the shearing stress and the heat flux, respectively. By using dimensionless variables and similarity transformations, we have

$$C_{fx}\, Gr^{1/4} = x\, f''(x,0) \text{ and } Nu_x\, Gr^{-1/4} = -x\theta(x,0). \tag{15}$$

Results and Discussion

The aim of this study is to investigate the effect of radiation on free convection flow for a circular cylinder in the presence heat generation and heat conduction. In the numerical computation, the value of the Prandtl number is taken as *Pr* = 0.70, which corresponds to air and various values of the remaining parameters are used. To assess the accuracy of the present results, the results for the heat transfer without radiation and heat generation effect were compared with the previously published results and a good agreement were found among these three observations. Moreover, the boundary condition $y \to \infty$ is approximated as $y_{\max} = 8$ which is sufficiently large for the velocity to approach the relevant free stream velocity.

Table 1. Comparison of the present numerical results of the heat transfer rate with Prandtl number *Pr* = 1.0 and *p* = 1.0

x	Merkin (1976)	Nazar *et al.* (2002)	Hye *et al.* (2007)	Present (2014)
0.00	0.4214	0.4214	0.4241	0.4216
$\pi/6$	0.4161	0.4161	0.4161	0.4163
$\pi/3$	0.4007	0.4005	0.4005	0.4007
$\pi/2$	0.3745	0.3741	0.3741	0.3742
$2\pi/3$	0.3364	0.3355	0.3355	0.3356
$5\pi/6$	0.2825	0.2811	0.2811	0.2811
π	0.1945	0.1916	0.1916	0.1912

The quantities of physical significance are the velocity and temperature distribution as well as the skin friction and heat transfer factors which are obtained from the numerical solution of the equations (11) and (12) together with the moderated boundary conditions (13) by the finite difference method. The numerical solutions start at the lower stagnation point (at $x = 0$) of the cylinder and proceed round the cylinder up to the upper stagnation point (at $x = \pi$). Then the resulting solutions for the velocity and temperature are presented in Figs. 2 and 4 and the local skin friction and heat transfer are depicted in Figs. 3 and 5.

The effect of thermal radiation parameter on the velocity and temperature profiles is illustrated in Fig. 2 while the value of the heat generation parameter is taken as $Q = 0.01$. Figs. 2a and 2b, reveal that the dimensionless velocity, u and the dimensionless temperature, θ increase as the value of Ra increases. This is because, greater Ra produce an appreciable increase in the temperature distribution within the boundary layer of the fluid due to the radiative heat transfer from the heat of the body to the fluid. Moreover, near the surface of cylinder the velocity increases significantly along y direction and become maximum then decreases asymptotically. The maximum values of the velocity are 0.45316, 0.46650, 0.48823, 0.50245 and 0.51434 for Ra = 0.01, 0.04, 0.08, 0.10 and 0.12, respectively. Comparing the peak values of the velocity, due to the change of Ra from 0.01 to 0.12 the velocity increases by 13.50 %. The numerical results of the local skin friction and heat transfer rate for the selected values of thermal radiation parameter while $Pr = 0.70$ and $Q = 0.01$ are plotted in Figs. 3a and 3b, respectively. From these figures, it can be seen that an increase in the radiation parameter leads to increase the skin friction factor and decrease the local heat transfer rate. This outcome is consistent with the velocity and temperature behavior close to surface of the cylinder as pointed out in Figs. 2a and 2b, because the greater Ra creates a typical hot fluid layer near the surface of the cylinder which turns in lower temperature difference between the solid and the ambient fluid.

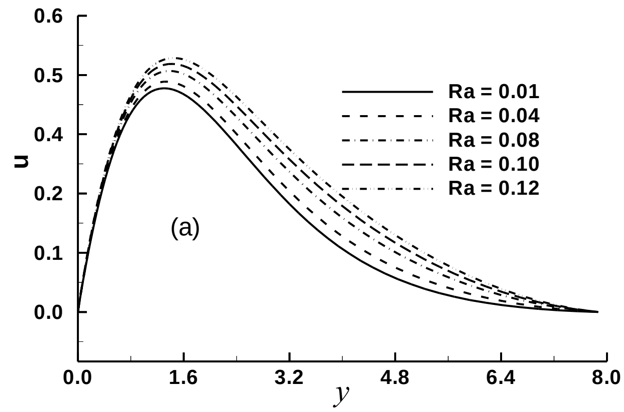

Fig. 2a. Variation of velocity against *y* for varying of *Ra* with *Pr* = 0.70 and *Q* = 0.01.

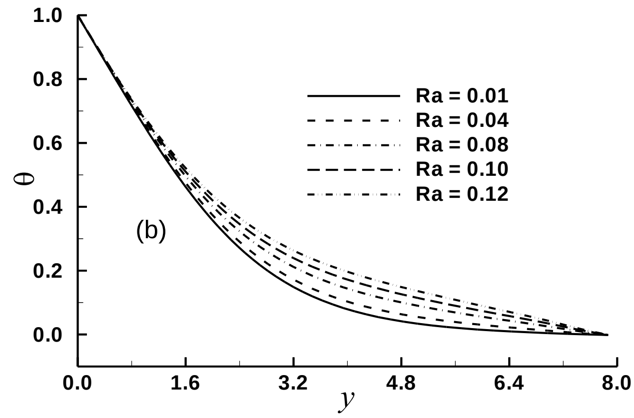

Fig. 2b. Variation of temperature against *y* for varying of *Ra* with *Pr* = 0.70 and *Q* = 0.01.

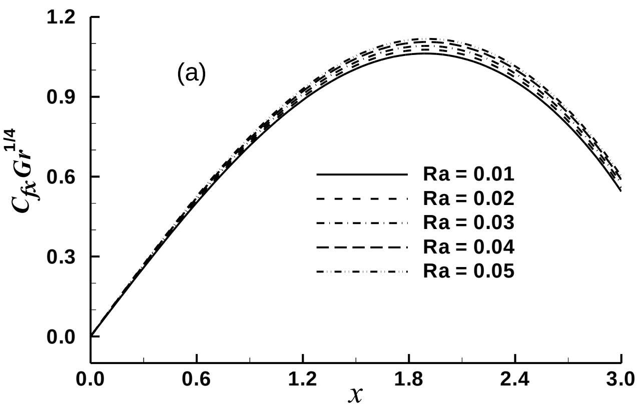

Fig. 3a. Variation of skin friction against *x* for varying of *Ra* with *Pr* = 0.70 and *Q* = 0.01.

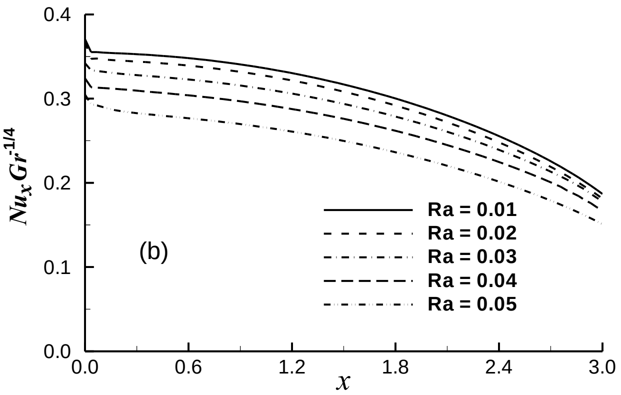

Fig. 3b. Variation of heat transfer rate against x for varying of Ra with Pr = 0.70 and Q = 0.01.

Moreover, it is significant from the practical point of view that the skin friction increases by 5.17 % as Ra increases from 0.01 to 0.05.

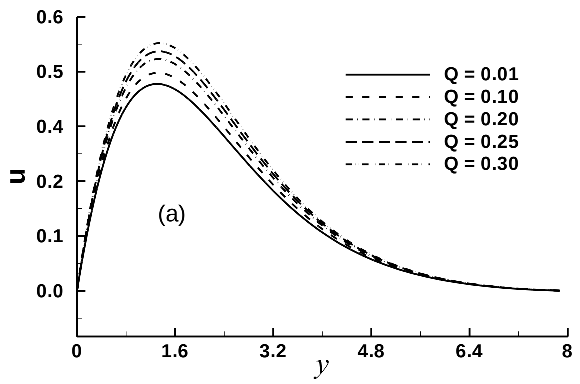

Fig. 4a. Variation of velocity against y for varying of Q with Pr = 0.70 and Ra = 0.01.

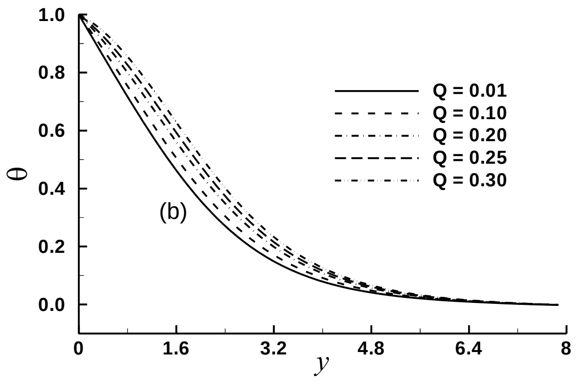

Fig. 4b. Variation of temperature against y for varying of Q with Pr = 0.70 and Ra = 0.01.

The variation of the velocity and temperature profile for the various values of heat generation parameter at $x = \frac{\pi}{6}$ having Prandtl number Pr = 0.70 with Ra = 0.01 is depicted in Fig. 4 (a) and 4(b), respectively. It can be observed that, both the velocity and temperature increase with the increasing value of *Q*. Moreover, the velocity profile produced by heat generation parameter has a local maximum value and which are recorded as 0.45316, 0.47741, 0.50757, 0.52443 and 0.54232. From this observation, velocity increases by 19.68 % as *Q* increases from 0.01 to 0.30. On the other hand, Figs. 5(a) and 5(b), illustrate the effect of the internal heat generation parameter on the local skin friction and the local heat transfer rate.

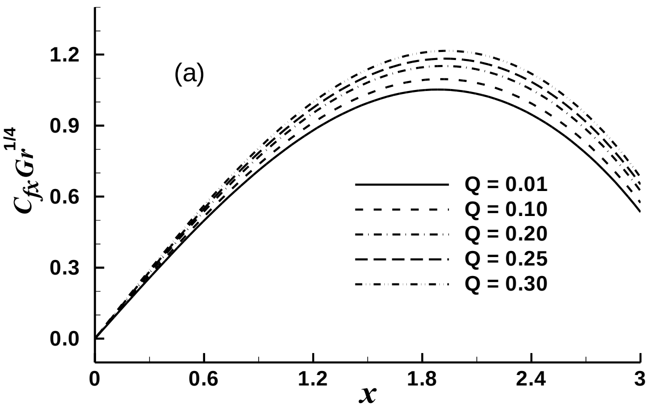

Fig. 5a. Variation of skin friction against *x* for varying of *Q* with *Pr* = 0.70 and *Ra* = 0.01.

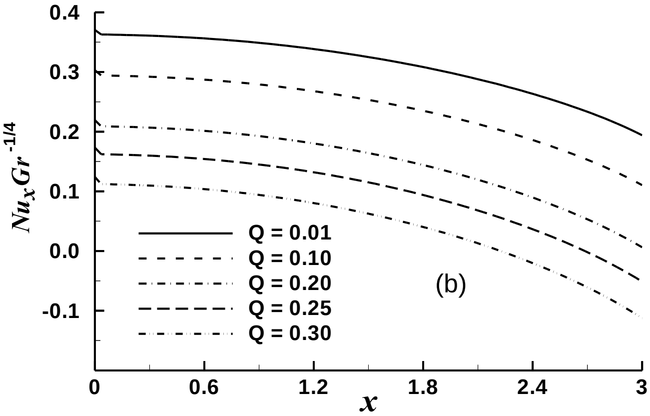

Fig. 5b. Variation of heat transfer rate against *x* for varying of *Q* with *Pr* = 0.70 and *Ra* = 0.01.

It is clear that the skin friction factor gradually increases but the opposite phenomenon takes place in the case of heat transfer rate. The trend is evident, since the increasing *Q* increases the fluid temperature as well as fluid motion within the boundary layer.

Conclusion

In this analysis, the effect of radiation on natural convection flow in presence of internal heat generation over a cylinder has been investigated. From the present investigation, it can be concluded that both radiation and heat generation have the effect to increase the fluid velocity and temperature, and accordingly, the skin friction factor increases but heat transfer rate decreases for the increasing of *Ra* and *Q*.

References

Akhter, R., M.M. Ali, M.B. Hossain and M.S. Uddin, (2013). Conjugate effect of radiation and thermal conductivity variation on MHD free convection flow for a vertical plate. *American Journal of Computational Mathematics*, **3:** 252-259.

Cebeci, T. and P. Bradshow, (1984). Physical computational aspects of convective heat transfer, *Springer*, New York.

Chamkha, A.J. and M.M.A. Quadri, (2001). Heat and mass transfer from a permeable cylinder in a porous medium with magnetic field and heat generation/ absorption effects. *Numer. Heat Transfer., A Appl.,* **40:** 387-401.

Cheng, C. Y., (2006). Free convection heat and mass transfer from horizontal cylinder of elliptic cross section in micropolar fluids. *International Communications in Heat and Mass Transfer*, **33:** 311-318.

El-Naby, M.A., .E. Elsayed, MN. Elbarbaryand and Y. Abdelazem, (2003). Finite difference solution of radiation effect on MHD unsteady free convection flow over a vertical plate variable surface temperature. *Journal of Applied Mathematics*, **2:** 65-86.

Hossain, M.A., M.A. Alim and D.A.S. Ress, (1998). Effect of thermal radiating on natural convection circular cylinders of elliptic cross section. *Acta Mechanica*, **129:** 171-186.

Hossain, M.A., K. Khanafer and K. Vafai, (2001). The effect of radiation on free convection flow of fluid with variable viscosity from a porous vertical plate. *International Journal of Thermal Science*, **40:** 115-124.

Hye, M.A., M.M. Molla and M.A.H. Khan, (2007). Conjugate effects of heat and mass transfer on natural convection flow across an isothermal horizontal circular cylinder with chemical reaction. *Nonlinear A nalysis: Modeling and Control*, **12:** 191-201.

Keller, H.B., (1978). Numerical methods in the boundary layer theory. *Annu. Rev. Fluid Mech.*, **10:** 417-433.

Mamun, A.A., Z.R. Chowdhury, M.A. Azim and M.M. Molla, (2008). MHD- conjugate heat transfer analysis for avertical flat plate in presence of viscous dissipation and heat generation. *International Communications in Heat and Mass Transfer*, **35:** 1275-1280.

Merkin, J.H., (1976). Free convection boundary layer on an isothermal horizontal circular cylinder: In ASME /AIChE, Heat Transfer Conference, St. Louis, Mo. August, pp. 9-11.

Molla, M.M., M.A. Hossain and R.S.R. Glora, (2005). Natural convection flow from an isothermal horizontal circular cylinder with temperature dependent viscosity. *Heat Transfer*, **41:** 594-598.

Mukhopadhyay, S., (2009). Effect of radiation and variable fluid viscosity on flow and heat transfer along a symmetric wedge. *Journal of Applied Fluid Mechanics*, **2:** 29-34.

Nazar, R., Amin, N., T. Grosan and I. Pop, (2002). Free convection boundary layer on an isothermal sphere in a micro polar fluid. *International Communications in Heat and Mass Transfer*, **29:** 377-386.