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OPTIMIZATION OF MULTI-OBJECTIVE TRANSPORTATION PROBLEM USING FUZZY LINEAR MEMBERSHIP FUNCTION

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Abstract

This paper presents the solution procedure of a multi-objective transportation problem (MOTP) using fuzzy linear membership function. At first, the problem has been transformed into a classical multi-objective transportation problem, where to minimize transportation cost, transportation time and damage. Then introducing the fuzzy linear membership function, the crisp model becomes a simple linear programming model. The overall satisfaction level for the decision-maker will be calculated. Real-life numerical example has been provided to illustrate the solution procedure for two and three objectives. The linear programming model was solved using LINGO software for operations research.

Keywords: Multi-objective transportation, Fuzzy programming technique, Membership function, Crisp model.

Introduction

The problems commonly discuss the distribution of goods from various supply points to different destination points are known as the transportation problem. Transportation problems are generally two types, single objective and multi-objective transportation problem. The traditional single objective transportation problem is a special type of linear programming problem and the constraints follow a particular mathematical structure. The source parameter (a_i) may be production facilities, warehouse etc. And the distribution parameter (b_j) may be warehouse, sales outlets etc. Besides, there is a penalty (c_{ij}), the coefficient of the objective function, associate with transporting a unit of product from source *i* to destination *j,* could represent the transportation cost, delivery time, number of goods transported, damage cost, unfulfilled demand and many more. Lee and Moore (1973) studied the optimization of transportation problems with multiple objectives. Diaz (1979) and Isermann (1979) developed different algorithms for all the non-dominated solutions for linear multi-objective transportation problems. Ringuest and Rinks (1987)

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developed two interactive solution algorithms for the linear multi-objective transportation problem.

Bit, Biswal, and Alam (1992) applied the fuzzy programming technique to solve the Multi-Objective Transportation Problem (MOTP) and obtained efficient solutions as well as an optimal compromise solution using a linear membership function. Verma, Biswal, and Biswas (1997) applied a fuzzy programming technique to solve the MOTP with some nonlinear membership functions. Recently, Zangiabadi and Malki (2007, 2013) used the Fuzzy Goal Programming (FGP) approach to solve MOTP with linear and nonlinear membership functions. Yaghoobi and Tamiz (2007) proposed an extension of the FGP approach. Kumar and Pandey (2012) solved MOTP using three different fuzzy programming methods.

Materials and Methods

The mathematical model of a Multi-objective Transportation Problem (MOTP) is written as follows: Minimize: $Z^k(x)=\sum_{i=1}^{m}\sum_{j=1}^{n}C_{ij}x_{ij}$

Subject to the constraints: $\sum_{j=1}^{n}x_{ij}=a_i, \quad i=1,2,3,\ldots\ldots\ldots\ldots,m$

$$\sum_{i=1}^{m}x_{ij}=b_j, \quad j=1,2,3,\ldots\ldots\ldots\ldots,n$$

$$\forall\ x_{ij}\geq 0, \quad i=1,2,\ldots\ldots m; \quad j=1,2,\ldots\ldots,n$$

and $\sum_{i=1}^{m}a_i=\sum_{j=i}^{n}b_j$ balance condition.

Fuzzy Programming Technique to solve MOTP

To solve the MOTP in a fuzzy programming technique, at first, find the lower bound L_k and the upper bound U_k of the k^{th} objective function Z_k, $k=1,2,\ldots.,K$ where U_k is the height acceptable level of achievement for k^{th} objective, L_k is the aspiration level of achievement for k^{th} objective and $d_k=U_k-L_k$ is the degradation allowance for the k^{th} objective. When the aspiration level for each objective has specified, a fuzzy model is formed and then the fuzzy model is converted into a crisp model. The solution for MOTP using fuzzy technique required the following steps:

Step 1: Solve the MOTP as a single objective transportation problem.

Step 2: From the above result, determine the corresponding values of each objective function and then arrange a payoff matrix as follows:

	$Z_1(X)$	$Z_2(X)$	. . .	$Z_K(X)$
$X^{(1)}$	Z_{11}	Z_{12}	. . .	Z_{1K}
$X^{(2)}$	Z_{21}	Z_{22}	. . .	Z_{2K}
.	.	.	. . .	.
.	.	.	. . .	.
.	.	.	. . .	.
$X^{(K)}$	Z_{K1}	Z_{K2}	. . .	Z_{KK}

Where $X^{(1)}, X^{(2)}, \ldots\ldots\ldots, X^{(K)}$ are the optimal solutions and $Z_{1K}, Z_{1K}, \ldots\ldots\ldots, Z_{KK}$ are the corresponding value of the single objective function.

Step 3: From step 2, find for every single objective, the U_k and the corresponding L_k for each solution set, where U_k = maximum $\left(Z_{1K}, Z_{2K}, \ldots\ldots\ldots, Z_{KK}\right)$ and L_k = minimum $\left(Z_{1K}, Z_{2K}, \ldots\ldots\ldots, Z_{KK}\right)$, $k = 1,2,\ldots., K$. Then an initial fuzzy model can find as follows:

Find X_{ij}, $i = 1,2,3,\ldots.., m$, $j = 1,2,3,\ldots, n$;

$Z_K \leq L_K$, $k = 1,2,\ldots., K$

Subject to the constraints: $$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2, 3, \ldots\ldots\ldots\ldots, m$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, 2, 3, \ldots\ldots\ldots\ldots, n$$

$$\forall\ x_{ij} \geq 0, \quad i = 1,2,\ldots\ldots m; \quad j = 1,2,\ldots\ldots., n$$

Step 4: Define a membership function $\mu(Z_k)$ for the k^{th} objective function.

Step 5: Convert the fuzzy model, in step 3, as a crisp model as follows:

Maximize β

Subject to the constraints: $$\beta \leq \mu(Z_k)$$

$$\sum_{j=1}^{n} X_{ij} = a_i, \quad i = 1, 2, 3, \ldots\ldots\ldots\ldots, m$$

$$\sum_{i=1}^{m} X_{ij} = b_j, \quad j = 1, 2, 3, \ldots\ldots\ldots\ldots, n$$

$$\beta \geq 0,\ X_{ij} \geq 0, \quad i = 1,2,\ldots\ldots m; \quad j = 1,2,\ldots\ldots., n$$

Step 5: Solve the crisp model as a mathematical programming model.

Step 6: Optimal solution of the multi-objective transportation is obtained.

Fuzzy Linear Membership Function

A linear membership function is defined as :

$$\mu_k\left(Z^k(x)\right) = \begin{cases} 1, & \text{if} \quad Z_k \leq L_k \\ 1 - \dfrac{Z_k - L_k}{U_k - L_k} & \text{if} \quad L_k < Z_k < U_k \\ 0, & \text{if } Z_k \geq U_k \end{cases}$$

Using this linear membership function, the crisp model in step 5 can be model as a simple linear programming model as follows:

Maximize: β

Subject to the constraints: $Z_k + \beta(U_k - L_k) \leq U_k, \quad k = 1,2,\dots,K$

$$\sum_{j=1}^{n} X_{ij} = a_i, \quad i = 1, 2, 3, \dots\dots\dots\dots, m$$

$$\sum_{i=1}^{m} X_{ij} = b_j, \quad j = 1, 2, 3, \dots\dots\dots\dots, n$$

$$\beta \geq 0, \; X_{ij} \geq 0, \quad i = 1,2,\dots\dots m; \quad j = 1,2,\dots\dots, n$$

Results and Discussion

To test the feasibility of the fuzzy technique, let us consider the following problems where the decision-maker (DM) plans to distribute his productions from several facilities (origins) to different destinations (demands). During the planning, the DM wishes to optimize the following objective functions:

i. Minimize the transportation cost (Z^1)
ii. Minimize the transportation time (Z^2)
iii. Minimize the damage cost (Z^3)

The data for the transportation costs C^t for $t = 1, 2, 3$ are represented in dollar ($) for the following examples.

Example 1

A Company has three production facilities (Origins) X, Y, Z with a production capacity of 8, 19, 17 units of a product respectively. These units are to be shipped four warehouses A, B, C, and D with requirements (Demand) of 11, 3, 14, and 16 units respectively. The transportation cost and time between factories to warehouses are as follows:

$$C^1 = \begin{bmatrix} 1 & 1 & 5 & 4 \\ 1 & 7 & 2 & 3 \\ 7 & 7 & 3 & 5 \end{bmatrix} \text{and } C^2 = \begin{bmatrix} 3 & 2 & 2 & 1 \\ 4 & 7 & 7 & 9 \\ 4 & 1 & 3 & 1 \end{bmatrix}$$

The mathematical model of the problem is as follows

Minimize:

$$Z^1 = x_{11} + x_{12} + 5x_{13} + 4x_{14} + x_{21} + 7x_{22} + 2x_{23} + 3x_{24} + 7x_{31} + 7x_{32} + 3x_{33} + 5x_{34}$$

$$Z^2 = 3x_{11} + 2x_{12} + 2x_{13} + x_{14} + 4x_{21} + 7x_{22} + 7x_{23} + 9x_{24} + 4x_{31} + x_{32} + 3x_{33} + x_{34}$$

Subject to the constraints:

$$x_{11} + x_{12} + x_{13} + x_{14} = 8,$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 19$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 17$$
$$x_{11} + x_{21} + x_{31} = 11$$

$$x_{12} + x_{22} + x_{32} = 3$$
$$x_{13} + x_{23} + x_{33} = 14$$
$$x_{14} + x_{24} + x_{34} = 16$$

To find the optimum transportation cost and transportation time, at the first step, by using TORA software, the solution of each objective function of the transportation problem is
$X^1 = (x_{12} = 3, x_{14} = 5, x_{23} = 8, x_{24} = 11, x_{31} = 11, x_{33} = 6)$
$X^2 = (x_{13} = 6, x_{14} = 2, x_{21} = 11, x_{23} = 8, x_{32} = 3, x_{34} = 14)$ and
then $Z^1(X^1) = 110, \quad Z^2(X^1) = 216, \quad Z^1(X^2) = 131, \quad Z^2(X^2) = 131$

Thus we have membership functions are $110 \le Z^1 \le 131$ and $131 \le Z^2 \le 216$, fuzzy membership functions of both $Z^1(x)$ and $Z^1(x)$ are as follows:

$\mu_1(Z^1(x)) = \frac{131 - Z^1(x)}{131 - 110}$ and $\mu_2(Z^2(x)) = \frac{216 - Z^2(x)}{216 - 131}$

Now, the problem can be written as a linear programming model as follows:

Maximize: β

Subject to:

$$x_{11} + x_{12} + x_{13} + x_{14} = 8$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 19$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 17$$
$$x_{11} + x_{21} + x_{31} = 11$$
$$x_{12} + x_{22} + x_{32} = 3$$
$$x_{13} + x_{23} + x_{33} = 14$$
$$x_{14} + x_{24} + x_{34} = 16$$

$$0.00763x_{11} + 0.00768x_{12} + 0.05344x_{13} + 0.03053x_{14} + 0.00763x_{21} + 0.05344x_{22} + 0.01527x_{23} + 0.0229x_{24} + 0.05344x_{31} + 0.05344x_{32} + 0.0229x_{33} + 0.03817x_{34} + 0.16031\beta \le 1$$

$$0.01389x_{11} + 0.00926x_{12} + 0.00926x_{13} + 0.00463x_{14} + 0.01852x_{21} + 0.03241x_{22} + 0.03241x_{23} + 0.04167x_{24} + 0.01852x_{31} + 0.00463x_{32} + 0.01389x_{33} + 0.00463x_{34} + 0.39352\beta \le 1$$

Solving by LINGO, we have the following compromise solution $X^* = (x_{11} = 1, x_{12} = 3, x_{14} = 4, x_{21} = 10, x_{24} = 9, x_{33} = 14, x_{34} = 3)$ the objective values are $Z^1(X^*) = 114$, $Z^2(X^*) = 176$ and $\beta = 0.41782$
that is the cost for transportation is 114 units and time is 176 units and in that way, the satisfaction level for the decision-maker will be 41.78%.

Example 2

Let us consider another company has four production facilities (Origins) with a production capacity of 5, 4, 2, 9 units of a product respectively. These units are to be shipped five warehouses with requirements (Demand) of 4, 4, 6, 2 and 4 units respectively. The transportation cost $\left(C^1\right)$, transportation time $\left(C^2\right)$ and damage cost $\left(C^3\right)$ associate with the multi-objective transportation problem are as follows:

$$C^1=\begin{bmatrix}9&12&9&6&9\\7&3&7&7&5\\6&5&9&11&3\\6&8&11&2&2\end{bmatrix},\ C^2=\begin{bmatrix}2&9&8&1&4\\1&9&9&5&2\\8&1&8&4&5\\2&8&6&9&8\end{bmatrix}\text{and } C^3=\begin{bmatrix}2&4&6&3&6\\4&8&4&9&2\\5&3&5&3&6\\6&9&6&3&1\end{bmatrix}$$

The mathematical model of the problem is as follows

Minimize:

$$Z^1=9x_{11}+12x_{12}+9x_{13}+6x_{14}+9x_{15}+7x_{21}+3x_{22}+7x_{23}+7x_{24}+5x_{25}+6x_{31}+5x_{32}+9x_{33}+11x_{34}+ \\ 3x_{35}+6x_{42}+8x_{42}+11x_{43}+2x_{44}+2x_{45}$$

$$Z^2=2x_{11}+9x_{12}+8x_{13}+x_{14}+4x_{15}+x_{21}+9x_{22}+9x_{23}+5x_{24}+2x_{25}+8x_{31}+x_{32}+8x_{33}+4x_{34}+ \\ 5x_{35}+2x_{42}+8x_{42}+6x_{43}+9x_{44}+8x_{45}$$

$$Z^3=2x_{11}+4x_{12}+6x_{13}+3x_{14}+6x_{15}+4x_{21}+8x_{22}+4x_{23}+9x_{24}+2x_{25}+5x_{31}+3x_{32}+5x_{33}+3x_{34}+ \\ 6x_{35}+6x_{42}+9x_{42}+6x_{43}+3x_{44}+x_{45}$$

Subject to the constraints:

$$x_{11}+x_{12}+x_{13}+x_{14}+x_{15}=5$$
$$x_{21}+x_{22}+x_{23}+x_{24}+x_{25}=4$$
$$x_{31}+x_{32}+x_{33}+x_{34}+x_{35}=2$$
$$x_{41}+x_{42}+x_{43}+x_{44}+x_{45}=9$$
$$x_{11}+x_{21}+x_{31}+x_{41}=4$$
$$x_{12}+x_{22}+x_{32}+x_{42}=4$$
$$x_{13}+x_{23}+x_{33}+x_{43}=6$$
$$x_{14}+x_{24}+x_{34}+x_{44}=2$$
$$x_{15}+x_{25}+x_{35}+x_{45}=4$$

To find the optimum transportation cost, time and damage cost, at the first step, by using TORA software, the solution of each objective function of the transportation problem is

$X^1 = (x_{13} = 5,\ x_{22} = 3, x_{23} = 1, x_{31} = 1, x_{41} = 3, x_{44} = 2, x_{45} = 4)$
$Z^1(X^1) = 102,\quad Z^2(X^1) = 141,\quad Z^3(X^1) = 94$
$X^2 = (x_{11} = 3,\ x_{14} = 2, x_{25} = 4, x_{32} = 2, x_{41} = 1, x_{42} = 2, x_{43} = 6)$
$Z^1(X^2) = 164,\quad Z^2(X^2) = 72,\quad Z^3(X^2) = 90$
$X^3 = (x_{11} = 3,\ x_{12} = 2, x_{21} = 1, x_{23} = 3, x_{32} = 2, x_{43} = 3, x_{44} = 2, x_{45} = 4)$
$Z^1(X^3) = 134,\quad Z^2(X^3) = 122,\quad Z^3(X^3) = 64$

Thus we have membership functions are $102 \le Z^1 \le 164$, $72 \le Z^2 \le 141$ and $64 \le Z^3 \le 94$ and then fuzzy membership functions of $Z^1(x)$, $Z^2(x)$ and $Z^3(x)$ are as follows:

$\mu_1(Z^1(x)) = \frac{164 - Z^1(x)}{164 - 102}$, $\mu_2(Z^2(x)) = \frac{141 - Z^2(x)}{141 - 72}$ and $\mu_3(Z^3(x)) = \frac{94 - Z^3(x)}{94 - 64}$ respectively.

Now, the problem can be written as

Maximize: β

Subject to:

$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 5$$
$$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} = 4$$
$$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} = 2$$
$$x_{41} + x_{42} + x_{43} + x_{44} + x_{45} = 9$$
$$x_{11} + x_{21} + x_{31} + x_{41} = 4$$
$$x_{12} + x_{22} + x_{32} + x_{42} = 4$$
$$x_{13} + x_{23} + x_{33} + x_{43} = 6$$
$$x_{14} + x_{24} + x_{34} + x_{44} = 2$$
$$x_{15} + x_{25} + x_{35} + x_{45} = 4$$

$$\begin{aligned}&0.0549x_{11} + 0.0732x_{12} + 0.0549x_{13} + 0.0366x_{14} + 0.0549x_{15} + 0.0427x_{21} + 0.0183x_{22} + 0.0427x_{23}\\&+ 0.0427x_{24} + 0.0305x_{25} + 0.0366x_{31} + 0.0305x_{32} + 0.0549x_{33} + 0.0671x_{34} + 0.0183x_{35} + 0.0366x_{41}\\&+ 0.0488x_{42} + 0.0671x_{43} + 0.0122x_{44} + 0.0122x_{45} + 0.3780\beta \le 1\end{aligned}$$

$$\begin{aligned}&0.0142x_{11} + 0.0638x_{12} + 0.0567x_{13} + 0.0142x_{14} + 0.0284x_{15} + 0.0142x_{21} + 0.0638x_{22} + 0.0638x_{23}\\&+ 0.0355x_{24} + 0.0142x_{25} + 0.0576x_{31} + 0.0071x_{32} + 0.0567x_{33} + 0.0284x_{34} + 0.0355x_{35} + 0.0142x_{41}\\&+ 0.0567x_{42} + 0.0426x_{43} + 0.0638x_{44} + 0.0567x_{45} + 0.4894\beta \le 1\end{aligned}$$

$$0.0213x_{11}+0.0426x_{12}+0.0638x_{13}+0.0319x_{14}+0.0638x_{15}+0.0426x_{21}+0.0851x_{22}+0.0426x_{23}$$
$$+0.0957x_{24}+0.0213x_{25}+0.0532x_{31}+0.0319x_{32}+0.0532x_{33}+0.0319x_{34}+0.0638x_{35}+0.0638x_{41}$$
$$+0.0957x_{42}+0.0638x_{43}+0.0391x_{44}+0.0106x_{45}+0.3191\beta\leq 1$$

Solving by LINGO, we have the following compromise solution
$X^*=(x_{11}=3,\, x_{14}=2,\, x_{22}=2,\, x_{23}=2,\, x_{32}=2,\, x_{41}=1,\, x_{43}=4,\, x_{45}=4)$
the objective values are $Z^1(X^*)=119$, $Z^2(X^*)=104$, $Z^3(X^*)=76$ and $\beta=0.5557$
that is the cost for the transportation is 119 units, time required 104 units and damage cost is 76 units. The satisfaction level for the decision-maker will be 55.57%.

Conclusion

The present paper holds a solution procedure for the multi-objective transportation problem. The problem has been converted into a classical multi-objective transportation problem. Then the problem becomes a crisp problem. To obtain the solution of the transformed classical multi-objective transportation problem, the fuzzy programming technique has been used. The compromise solution and satisfaction level of the decision-maker were obtained using LINGO for operations research software.

References

A.K. Bit, M.P. Biswal and S.S. 1992. Alam, Fuzzy programming approach to multicriteria decision making transportation problem, *Fuzzy Sets and Systems,* North-Holland, 50:135-141

Abd El-Wahed, Waeil F., 2001. A multi-objective transportation problem under fuzziness, *Fuzzy Sets and Systems,* 117: 27–33.

Charnes, A., & Cooper, W. W., 1961. Management models and industrial applications of linear programming. New York, NY, Wiley.

Charnes, A., & Cooper, W. W., 1977. Goal programming and multiple objectives optimizations. *European Journal of Operational Research*, 1: 39–54.

Gupta, N., Ali, I., & Bari, A., 2013. A compromise solution for multiobjective chance constraint capacitated transportation problem. *Prob Stat Forum*, 26: 60–67.

Kumar, S., & Pandey, D., 2012. Fuzzy programming approach to solve multi-objective transportation problem. *Proceedings of International Conference on Soft Computing for Problem Solving*, 130: 525–533.

Lee, S. M., & Moore, L. J., 1973. Optimizing transportation problems with multiple objectives. AIIE transactions, 5(4): 333–338.

Lee, S. M., & Olson, D. L., 1999. Goal programming, multicriteria decision making: Advances in MCDM models, algorithms, theory, and applications. Boston, MA: Kluwer Academic.

Maity, G., & Roy, S. K., 2014. Solving multi choice multi objective transportation problem: a utility function approach. *Journal of Uncertainty Analysis and Applications,* 2, 11.

Mohammad Ali Yaghoobi and Mehrdad Tamiz, 2007. A method for solving fuzzy goal programming problems based on MINMAX Approach, *European Journal of Operational Research* 177(3):1580-1590

Quddoos, A., Javaid, S., & Khalid, M. M., 2013b. A new method to solve Bi-objective transportation problem. *International Journal of Applied Mathematics,* 26: 555–563.

Quddoos, A., Javaid, S., Ali, I., & Khalid, M. M., 2013a. A Lexicographic goal programming approachfor a bi-objective transportation problem. *International Journal of Scientific & Engineering Research,* 4, 7: 1084– 1089.

Rakesh Verma, M.P. Biswal, 1997. A. Biswas, Fuzzy programming technique to solve multi-objective transportation problems with some non-linear membership functions, *Fuzzy Sets and Systems*, 91: 37-43

Zangiabadi, M., & Maleki, H. R., 2007. Fuzzy goal programming for multiobjective transportation problems. *Journal of Applied Mathematics and Computing*, 24: 449–460.

Zangiabadi, M., & Maleki, H. R., 2013. Fuzzy goal programming technique to solve multiobjective transportation problems with some non-linear membership functions. Iranian Journal of Fuzzy Systems, 10: 61–74.

Zeleny, M.,1982. Multi criteria decision making. *McGraw-Hills*, New York, NY.